



The Open University

MU120
Open Mathematics

Resource Book A

Units 1–5



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The Open University, Walton Hall, Milton Keynes, MK7 6AA.

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Introduction

This is the first of four resource books which are part of the MU120 course materials. There is one book corresponding to each of the main blocks of the course: A, B, C and D.

The resource books are intended to give you further practice, if you require it, at using some of the major mathematical ideas from the course. Most of the questions can be used by you individually to practise basic skills. Others are best tried in small groups, perhaps in tutorials or by self-help groups.

You will find that questions in the resource books give appropriate references to the course units or the *Calculator Book*. However, the course units will *not* usually direct you to the practice examples in the resource books—rather, you will need to decide which of the examples are helpful. You are certainly *not* expected to attempt all the questions. There are full solutions to each question and you may well find that the solutions are helpful if you are having problems with particular techniques. Occasionally you will find that the examples in the resource books are very similar to TMA or CMA questions.

This first book, *Resource Book A*, covers the major ideas in *Units 1 to 5* and corresponding sections in the *Calculator Book*. The questions are arranged in five sections corresponding to the five units and each question has a reference to the appropriate section of the unit or *Calculator Book*.

Unit 1 Mathematics everywhere

Question 1

Calc Book Sec. 1.5

Complete the table below.

	Original price	Percentage change	Final price
(a)	£1000	2% increase	
(b)	£500	15% decrease	
(c)	£6.50	25% increase	
(d)	£20.99	20% decrease	
(e)	£15 675	5% increase	
(f)	£4600	1.2% decrease	
(g)	£0.63	110% increase	
(h)	£23.15	100% decrease	

Question 2

Calc Book Sec. 1.5

Which of the following items has shown the greatest percentage price rise between 1994 and 2001?

		Price in 1994	Price in 2001
(a)	Pint of milk	36 pence	44 pence
(b)	Small car	£7990	£10 325
(c)	10-minute phone call	60 pence	55 pence
(d)	A basic radio	£6.99	£9.79

Question 3

Calc Book Sec. 1.5

- (a) What is 20% of £30?
- (b) Suppose you have inherited a 15% share of £1000 in a relative's will. How much would you inherit?
- (c) The harvest of wheat on a farm increases by 50% in one year. If the harvest last year was 50 000 bushels, what would the harvest be this year?
- (d) The price of a computer excluding VAT is £999. Assuming that VAT is 17.5%, calculate the total cost of the computer (to the nearest penny).

Question 4

Calc Book Sec. 1.5

Eleven people club together to buy equal shares in six lottery tickets. The tickets cost £1 each.

- (a) How much does each person have to pay?
- (b) What percentage of the winnings is each person entitled to?
- (c) Assuming that the first prize is £8 000 000, how much would each person win if one of the tickets won this prize?

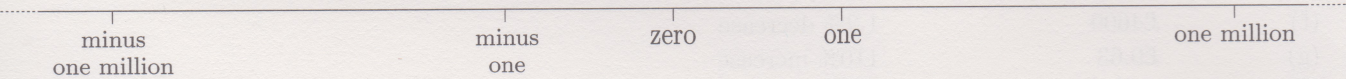
Question 5

Calc Book Sec. 1.6

Your calculator uses scientific notation to display numbers that do not otherwise fit on the screen. Test your awareness of this notation by placing the following numbers approximately onto the number line shown.

Note: the number line has not been drawn to scale.

1.0	7×10^{-7}	-2000
100 000	0.001	1E5
1×10^7	-3×10^5	-5×10^{-2}



Question 6

Calc Book Sec. 1.6

- (a) According to the World Wildlife Fund's *Atlas of the Environment*, the world's population at the start of 1992 was 'about 5.4 billion and it is expanding rapidly. Every day we share the earth and its resources with over 250 000 more people than the day before'. Assuming that the daily increase remains constant, in which year would the world population reach 6×10^9 people?
- (b) The calculation in part (a) was based upon two United Nations estimates: 5.4 billion and 250 000 and both are correct to two significant figures. Consider how these estimates might have been arrived at. Repeat the calculation to see the effect of using the following alternative numbers.
- (i) Assume that 5.4 billion and 250 000 had both been rounded, but that the first two digits were correct. This means that the population at the start of 1992 was between 5.35 and 5.45 billion, and the daily increase was between 245 000 and 255 000. What would be the longest time taken for the world population to reach 6×10^9 ?
- (ii) Assume that 5.4 billion had been rounded as in part (i) but 250 000 had been rounded to the nearest 50 000, i.e. nearer 250 000 than 200 000 or 300 000. What is the longest time for the world population to reach 6×10^9 ?
- (c) On 11 October 1999, the news media announced that at some point the following day, according to United Nations best estimates, a new-born baby would take the number of humans on the planet to six billion. It was said 'the child is most likely to be a boy born into a poor family in a rural part of north India or China'. What does this news indicate about the estimates that were being made in 1992 and 1999?

Question 7*Calc Book Sec. 1.6*

An ornithologist has estimated that every year about 50 billion birds make migratory journeys. A statement like that makes one wonder how such an estimate could possibly be made, as well as how accurate it is likely to be.

Consider each of the following investigations and decide what data you would need in order to make a reasonable estimate. How could you collect the data? (In some cases your estimate may be little more than an intelligent guess!) What assumptions would you need to make? How could you simplify the situation in order to make a start? Then try to work out some of these huge numbers. Like many investigations there is no clear-cut right answer and, in each case, you have to decide what the questions mean. For example, in part (a) you need to decide which beach and what its dimensions are.

(a) *Little grains of sand ...*

How many grains of sand are there on the beach?

(b) *Needles in a ...*

How many needles might there be in a forest of Norway Spruce trees (the traditional Christmas tree)?

(c) *Raindrops keep falling ...*

How many raindrops fall on the UK each year?

Question 8*Calc Book Sec. 1.7*

Use the MATH menu of your calculator to work out the value of each of these, correct to four decimal places.

- (a) 12.601^3 (b) $\sqrt[3]{10}$ (c) $\sqrt[4]{10}$ (d) $\sqrt[5]{10}$
 (e) $\sqrt[3]{7^3}$ (f) $(\sqrt[3]{7})^3$ (g) $\sqrt[12]{4096}$ (h) $\sqrt[6]{36^3}$
 (i) $\sqrt[3]{27}$ (j) $\sqrt[6]{27^2}$ (k) $\sqrt[3]{\sqrt[4]{4096}}$ (l) $\sqrt[4]{\sqrt[3]{4096}}$

Question 9*Calc Book Sec. 1.7*

Every 100 minutes, a weather satellite completes a circular polar orbit around the earth, flying 850 km above the surface.

- (a) If the radius of the earth is 6368 km, how far does the satellite travel every orbit? Give your answer to the nearest 1000 km.
 (The circumference of a circle can be calculated by multiplying its radius by 2π .)
 (b) How fast, in km per hour, is the satellite travelling? Give your answer correct to 2 s.f.

Question 10*Calc Book Sec. 1.7*

Use the Frac option in MATH menu of your calculator to change each of these decimals to fractions.

- (a) .125 (b) .24 (c) .995 (d) .111
 (e) .175 (f) .256 (g) .004 (h) 1.25

Question 11

Calc Book Sec. 1.7

- (a) Write down what happens when you press the following calculator key sequence. (Three dots are used to indicate that the sequence continues in the same way.)

9 ENTER

+ 4 ENTER ENTER ENTER ENTER ENTER ENTER ...

- (b) For each of the four sequences of numbers below:

- (i) describe how each number is obtained from the previous number;
 (ii) write down a calculator key sequence, similar to the one in part (a), which will generate the sequence.

A 20 18 16 14 12 10 ...

B 5 15 45 135 405 1215 ...

C 10 .1 10 .1 10 .1 ...

D 1 3 7 15 31 63 ...

Unit 2 Prices

Question 1

Unit 2 Sec. 1.1

In Unit 2, the price of a loaf of bread is used as an indicator of general price rises in England over the last four centuries. List other commodities that could have been used instead, and comment on the suitability of each. Do you think that bread is a good choice?

Question 2

Unit 2 Sec. 1.1

From a food budget of £65 per week, a family spends £4.80 on bread, £7.60 on milk and £15 on fresh fruit and vegetables. Calculate the percentage of the food budget spent on (a) bread, (b) milk and (c) fresh fruit and vegetables. Give your answers rounded to one decimal place.

Question 3

Unit 2 Sec. 1.1

Find the percentage increase or decrease in the average price of each of the items in the table below between January 2003 and January 2004. Give your answers rounded to one decimal place.

Item		Average price (pence)	
		January 2003	January 2004
(a)	Tomatoes (per kg)	131	142
(b)	Cucumber (each)	67	77
(c)	Bananas (per kg)	95	86
(d)	Unleaded petrol (per litre)	76	76
(e)	Self-raising flour (per 1.5 kg)	59	66
(f)	Pork sausages (per kg)	326	318
(g)	Tea bags (per 250 g)	150	141

Question 4

Unit 2 Sec. 3.1

Find the mean and the median of each of the following batches of data.

- (a) 7 7 9 6 4 10 8 12
- (b) 23 21 26 26 24
- (c) 101 107 98 92 115 102
- (d) 74 38 57 93 47 64 71

Question 5

Unit 2 Sec. 3.2

The table below gives information on the number of brothers and sisters of the children in a primary school class. Find the mean number of brothers and sisters of the children in the class.

Number of siblings	Frequency
0	7
1	18
2	5
3	2
4	0
5	1

Question 6

Unit 2 Sec. 3.2

The amounts of petrol bought by a motorist in a month in 2005 and the price paid per litre are given in the table below.

Amount of petrol (litres)	Price per litre (pence)
24.3	85.1
15.8	86.9
26.4	84.9
22.0	85.6

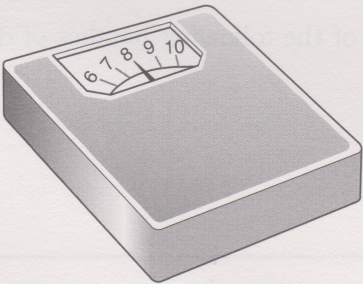
Find the mean price per litre paid by the motorist.

Question 7

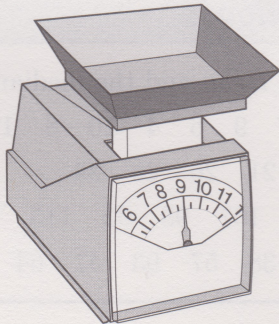
Unit 2 Sec. 3.2

A father attempted to weigh his baby. Unfortunately, the baby would not lie still. The father weighed the baby on both kitchen scales and bathroom scales. On the kitchen scales the readout was 9 kg and on the bathroom scales the readout was 8 kg. Neither of these results was very reliable, however, since the needle was waving about a lot!

The father wished to combine these two measurements to try and get a better estimate. The average weight of the baby was 8.5 kg, but this failed to take into account the fact that the kitchen scales were more accurate.



Bathroom scales



Kitchen scales

A way of combining the two measurements (and errors), which is quite often used with scientific data, is to calculate the weighted mean, where the weighting is given by the reciprocal of the estimated error. The father estimated that the error in the readout of the kitchen scales was $\frac{1}{2}$ kg in either direction, whereas that for the bathroom scales was 1 kg in either direction. So the weightings were $1 \div \frac{1}{2} = 2$ and $1 \div 1 = 1$ respectively. The data are shown in the table on the facing page.

	Weight of baby (kg)	Error in scales (kg)	Weight given to measurement
Kitchen scales	9	$\frac{1}{2}$	2
Bathroom scales	8	1	1

Calculate the weighted mean of the weight of the baby.

Question 8**Unit 2 Sec. 4.2**

The table below shows the weights given to the five items in the shopping basket of Section 4 of *Unit 2*. Also shown are the price ratios of the items for January 2003 relative to January 2002 and for January 2004 relative to January 2003. Find the basket price ratio (a) for January 2003 relative to January 2002 and (b) for January 2004 relative to January 2003.

Item	Price ratio for Jan. 2003 relative to Jan. 2002	Price ratio for Jan. 2004 relative to Jan. 2003	Weight
Large loaf	1.075	1.088	220
Milk	1.000	1.000	420
Eggs	0.988	1.012	50
Potatoes	0.898	1.057	55
Sugar	1.083	1.138	25

Question 9**Unit 2 Sec. 4.3**

The average January price for one pint of milk is given in the table below for each year from 1997 to 2004.

Year	1997	1998	1999	2000	2001	2002	2003	2004
Price (pence)	36	35	34	34	35	37	37	37

- Calculate the percentage increase in the average price of a pint of milk between January 1997 and January 2004.
- Calculate the percentage increase in the average price of a pint of milk between January 1998 and January 2002.
- Calculate the price ratio relative to January 1997 for each January from 1998 to 2004. Give your answers correct to 3 d.p.
- Take 1997 as the base year and calculate the value of the price index for a pint of milk for each January from 1997 to 2004. Give your answers correct to 1 d.p.
- Use the price index calculated in part (d) to calculate the percentage increase or decrease in the average price of a pint of milk, giving answers correct to the nearest whole number, over the following intervals.
 - Between January 1998 and January 2002.
 - Between January 1998 and January 2000.
- Use the previous-year method on the data in the table to calculate values of a price index for a pint of milk. Take 1997 as the base year and calculate the value of the index each January from 1997 to 2004 rounded to 1 d.p.
- Are there any discrepancies between the values calculated using the base-year method in part (d) and those calculated using the previous-year method in part (f). How do you account for any discrepancies?

Question 10

Unit 2 Sec. 4.3

The average January price for a dozen large eggs (size 2) is given in the table below for each year from 1997 to 2004.

Year	1997	1998	1999	2000	2001	2002	2003	2004
Price (pence)	162	157	157	163	168	172	170	172

Answer the questions in parts (a)–(g) of Question 8 but for a dozen large eggs (size 2) instead of a pint of milk.

Question 11

Unit 2 Sec. 5.2

Find the value of the RPI in July 2004, in September 2004 and in October 2004 using the information in the table below and the fact that the value of the RPI in January 2004 was 183.1.

Group	Price ratio relative to January 2004			2004 weights
	July 2004	September 2004	October 2004	
Food and catering	0.997	0.998	1.001	160
Alcohol and tobacco	1.021	1.024	1.025	97
Housing and household expenditure	1.047	1.062	1.068	367
Personal expenditure	0.985	1.008	1.010	93
Travel and leisure	1.010	1.006	1.005	283

Question 12

Unit 2 Sec. 5

Between October 2003 and October 2004, the price of cheese rose on average by 0.2%, while the price of tobacco rose on average by 3.8%. Answer the following questions about the likely effects of these changes on the value of the RPI.

- (a) Would the change in the price of cheese be likely to lead to an increase or a decrease in the value of the RPI? Would the change in the price of tobacco be likely to lead to an increase or a decrease in the value of the RPI?
- (b) In each case, is the size of the increase or decrease likely to be large or small?
- (c) Using what you know about how RPI weights are produced, and your general knowledge about what people in the UK spend their money on, decide which of cheese and tobacco probably has the larger weight.
- (d) Which of the price changes is likely to have a larger effect on the value of the RPI? Briefly explain your answer.

Question 13**Unit 2 Sec. 6.1**

In this question use the list of values of the CPI and RPI in the table below.

Month	CPI 2002	CPI 2003	RPI 2002	RPI 2003
January	107.1	108.6	173.3	178.4
February	107.3	109.0	173.8	179.3
March	107.7	109.4	174.5	179.9
April	108.1	109.7	175.7	181.2
May	108.4	109.7	176.2	181.5
June	108.4	109.6	176.2	181.3
July	108.1	109.5	175.9	181.3
August	108.4	109.9	176.4	181.6
September	108.7	110.2	177.6	182.5
October	108.9	110.4	177.9	182.6
November	108.9	110.3	178.2	182.7
December	109.3	110.7	178.5	183.5

Source: www.statistics.gov.uk

The base date for the CPIs is 1996, and for the RPIs it is January 1987.

- (a) Find the annual rate of inflation, based on the CPI, in each of the months named. Round your answer to one decimal place.
 - (i) February 2003
 - (ii) May 2003
 - (iii) September 2003
 - (iv) November 2003
- (b) Throughout this part, round your answers to the nearest pound.
 - (i) An index-linked pension (linked to the RPI) was £100 per week after adjustment for inflation in April 2002. What should it be after adjustment for inflation in April 2003?
- (c) An index-linked pension (linked to the RPI) was £85 per week after adjustment for inflation in June 2002. What should it be after adjustment for inflation in June 2003?
 - (i) An index-linked pension (linked to the RPI) was £121 per week after adjustment for inflation in October 2002. What should it be after adjustment for inflation in October 2003?
 - (ii) An index-linked pension (linked to the RPI) was £94 per week after adjustment for inflation in May 2002. What should it be after adjustment for inflation in September 2003?
- (d) Throughout this part, base your answers on the RPI, and round them to the nearest penny.
 - (i) Find the purchasing power of the pound in March 2003 compared with one year earlier.
 - (ii) Find the purchasing power of the pound in September 2003 compared with one year earlier.
 - (iii) Find the purchasing power of the pound in November 2003 compared with May 2002.
 - (iv) Find the purchasing power of the pound in December 2003 compared with the base date, January 1987.

Question 14

Unit 2 Sec. 7.2

The ratio of the height of an adult to his/her head circumference is, on average, about 3. The height and head circumference of each of four adults are given below.

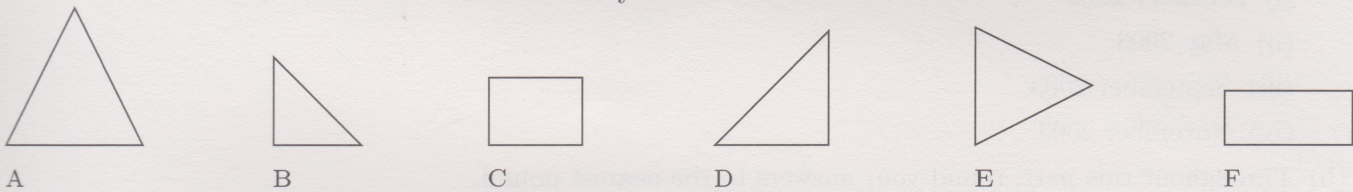
Name	Head circumference (cm)	Height (cm)
Abbas	56	164.5
Beryl	55	165.5
Carol	53.5	154
Derek	58.5	185

- (a) Which of the four people have a relatively large head circumference for their height?
- (b) Which of the four people have a relatively small head circumference for their height?
- (c) What, approximately, is the average head circumference of people who are 171 cm tall?

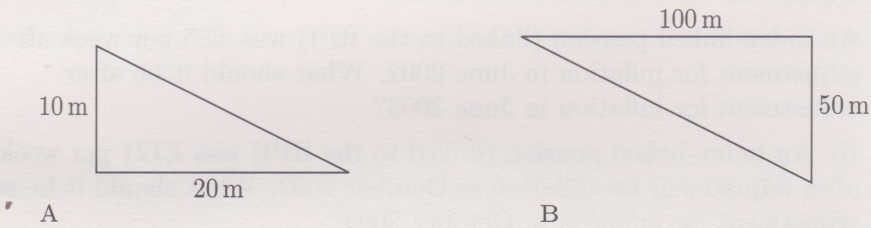
Question 15

Unit 2 Sec. 7.2

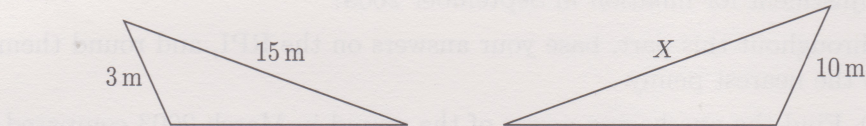
- (a) Look at the following diagrams. There are two pairs of similar shapes. Which are they?



- (b) The following two triangles are similar. What is the ratio of the sides of triangle B to the corresponding sides of triangle A?



- (c) Given that the following two triangles are similar, calculate the length of the side marked X.



Question 16

Unit 2 Sec. 7.2

Suppose you have a friend who wishes to buy a ladder that will reach to the top of the roof of her house. In order to estimate the height of the house, she looked at the picture below of a person standing outside the house. The particular person in the picture is nearly two metres tall. Estimate the height of the house (to the top of its roof).



Unit 3 Earnings

Question 1

Unit 3 Sec. 2.3 and Calc Book Sec. 3.2

For each of the following batches of data, calculate the mean and median of the data. Then draw a boxplot on your calculator and try to decide whether the mean or the median is more appropriate as a measure of the average of the batch of data.

- (a) 1 2 6 7 5 4 1 8
- (b) 1 2 3 2 1 3 1 1
- (c) 9 5 4 1 99 4 4 1
- (d) 1 30 20 55 3 78 21 63

Question 2

Calc Book Sec. 2.2 and 3.2

The daily midday temperatures in degrees Celsius ($^{\circ}\text{C}$) for June of a particular year are given below.

18 19 18 17 21 22 22 21 16 18 25 26 29 25 22
17 16 17 17 17 20 16 17 19 25 21 20 19 21 22

- (a) Sort the data into a frequency table, showing how many times each temperature occurred.
- (b) Enter the frequency table into your calculator and use it to calculate the mean and median of the temperatures.
- (c) Now draw a boxplot of the data on your calculator and from this consider whether the mean or the median is more appropriate as a measure of the average for this batch of data.

Question 3

Unit 3 Sec. 2.1

The table below gives the mean gross hourly earnings excluding overtime (in pence) of women and men working in Northern Ireland between 1998 and 2004.

	Women	Men
1998	752	906
1999	799	937
2000	829	970
2001	859	1013
2002	916	1044
2003	952	1091
2004	1014	1119

- (a) Calculate the earnings ratio at the mean for each year between 1998 and 2004. Give your answer as a percentage, correct to 1 d.p.
- (b) A similar earnings ratio was calculated in Activity 6 of *Unit 3* for the years from 1989 to 2003 for workers in Great Britain. It was concluded that there was evidence that the gap between women's and men's earnings had been closing slowly over that period. What pattern can you see from the data given above?

Question 4

Unit 3 Sec. 3.2

For each batch of data in Question 4 for *Unit 2* (on page 9 of this resource book), find the range, the quartiles and the interquartile range.

Question 5

Unit 3 Sec. 4 and 5

The table below gives the gross weekly earnings excluding overtime (in pounds) of 20 sales representatives (9 women and 11 men) in 2003.

Women	682	277	383	359	604	285	463	263	340		
Men	741	494	394	594	502	630	361	848	475	706	365

- (a) Use your calculator to obtain two boxplots of the sales representatives' earnings data, one for the women and one for the men.
- (b) Draw an accurate diagram of the boxplots.
- (c) What do the boxplots tell you about the relative earnings of male and female sales representatives?
- (d) What do the boxplots tell you about the shapes of the distributions of the earnings of male and female sales representatives?
- (e) Calculate the range and the interquartile range of the women's earnings and the men's earnings. Is the spread of the men's earnings greater or smaller than the spread of the women's earnings?

Question 6

Unit 3 Sec. 4 and 5

Repeat Question 5 for the earnings data in the table below, which gives the gross hourly earnings excluding overtime (in pence) of 17 postal workers, mail sorters, messengers and couriers (8 women and 9 men) in 2003.

Women	769	979	690	847	897	805	733	1088	
Men	800	1077	853	733	672	911	774	828	988

Question 7

Unit 3 Sec. 3

The table below gives the mean number of basic hours worked per week (excluding overtime) by male and female postal workers, mail sorters, messengers and couriers surveyed in the New Earnings Survey in 2003.

	Women	Men
Basic hours	36.8	37.2

For this occupational group, would the earnings ratio at the mean, calculated for gross weekly earnings (excluding overtime), be higher or lower than that calculated using gross hourly earnings (excluding overtime)?

Question 8

Unit 3 Sec. 5

The table below gives the distribution of gross hourly earnings excluding overtime (in pence) of postal workers, mail sorters, messengers and couriers surveyed in the New Earnings Survey in 2003.

	Women	Men
Highest decile	1005	1003
Upper quartile	920	908
Median	832	828
Lower quartile	762	771
Lowest decile	725	722

- (a) Draw decile boxplots for these data and use them to compare the earnings of male and female postal workers, mail sorters, messengers and couriers.
- (b) Calculate the earnings ratios at the median, the quartiles and the highest and lowest deciles for the postal workers, mail sorters, messengers and couriers. Give your answers as a percentage correct to the nearest whole number. Comment on what these figures tell you about the relative earnings of male and female workers in this occupational group.

Question 9

Unit 3 Sec. 5

The table below gives the distribution of gross weekly earnings excluding overtime (in pounds) of youth and community workers surveyed in the New Earnings Survey in 2003.

	Women	Men
Highest decile	514	541
Upper quartile	439	470
Median	374	378
Lower quartile	322	311
Lowest decile	268	260

- (a) Draw decile boxplots for these data.
- (b) Calculate the earnings ratios at the median, the quartiles and the highest and lowest deciles for the youth and community workers. Give your answers as a percentage correct to the nearest whole number.
- (c) Comment on what the boxplots and the earnings ratios tell you about the relative earnings of male and female workers in this occupational group.

Question 10

Unit 3 Sec. 6.1

- (a) The value of the AEI for May 2003 was 111.3 and its value for May 2004 was 115.3. Find the percentage increase in the value of the AEI between May 2003 and May 2004 (to 1 d.p.).
- (b) The value of the AEI for September 2003 was 112.8 and its value for September 2004 was 117.1. Find the percentage increase in the value of the AEI between September 2003 and September 2004 (to 1 d.p.).

Question 11**Unit 3 Sec. 6.2**

Values of the AEI and the RPI for various dates are given in the table below.

Date	RPI	AEI	Date	RPI	AEI
January 2003	178.4	109.9	January 2004	183.1	118.3
April 2003	181.2	110.7	April 2004	185.7	115.6
June 2003	181.3	111.5	June 2004	186.8	116.1
August 2003	181.6	112.2	August 2004	187.4	116.8

For each of the following months, calculate as a percentage the real earnings for that month compared with one year earlier (to 1 d.p.).

- (a) January 2004
- (b) April 2004
- (c) June 2004
- (d) August 2004

Unit 4 Health

Question 1

Calc Book Sec. 4.1

- (a) On your calculator, produce a scatterplot of the following paired data using suitable window settings.

First value	Second value
1.8	0.5
3.0	-0.9
-3.8	1.1
-3.0	-0.5

- (b) On your calculator, produce a scatterplot of the following paired data, using horizontal and vertical axes both ranging from 0 to 10. Draw the scatterplot on paper, labelling the axes appropriately.

First value	Second value
2	3
6	8
4	3
10	7
6	5
8	9
3	6

- (c) Use your calculator to help you draw on paper a scatterplot of the following data, choosing suitable axes.

Horizontal axis values	3	7	-1	-8	1	-5
Vertical axis values	0	-9	-2	6	9	1

Question 2

Calc Book Sec. 4.1

The table below records the goals scored for and against each of the ten teams in a school's football league.

Team	A	B	C	D	E	F	G	H	I	J
Goals for	8	12	6	14	7	18	5	10	11	12
Goals against	6	4	9	2	7	3	19	7	3	1

- (a) On your calculator, produce a scatterplot with 'goals for' on the horizontal axis and 'goals against' on the vertical axis.
- (b) Does the graph indicate any relationship between 'goals for' and 'goals against'?

Question 3

Unit 4 Sec. 1.3

The table below records the number of accidents per 1000 employees in a selection of seven factories.

Factory	A	B	C	D	E	F	G
Number of accidents per 1000 employees	4	6	16	2	10	8	4
Age of factory (years)	12	14	20	5	15	8	10

- (a) Draw on paper a scatterplot of these data, using your calculator to help you.

- (b) From the scatterplot, decide whether there appears to be a relationship between the age of a factory and the accident rate (that is, the number of accidents per 1000 employees) in the factory.

Question 4**Unit 4 Sec. 1.3**

The data in the table below are the percentage unemployed and the suicide rate per million in eleven cities in the USA in the 1970s.

City	Percentage unemployed	Suicide rate (per million)
New York	3.0	72
Los Angeles	4.7	224
Chicago	3.0	82
Philadelphia	3.2	92
Detroit	3.8	104
Boston	2.5	71
San Francisco	4.8	235
Washington	2.7	81
Pittsburgh	4.4	86
St Louis	3.1	102
Cleveland	3.5	104

- (a) Obtain a scatterplot for these data on the screen of your calculator with 'percentage unemployed' on the horizontal axis and 'suicide rate' on the vertical axis.
- (b) Draw a scatterplot of the data. Check your scatterplot against the one on your calculator screen. Label each point to indicate the city to which it corresponds.
- (c) Describe any conclusions you feel able to draw from the scatterplot.

Question 5**Unit 4 Sec. 1.3**

The data from two experiments to measure the expansion of water are given below. In each experiment the expansion volume and the temperature of 1 kg of water were measured. For each set of data, draw (on paper) a scatterplot and comment on the pattern of the data.

- (a) Temperature ($^{\circ}\text{C}$) Expansion (cm^3)

50	12
58	16
66	20
74	25
82	30
90	36

- (b) Temperature ($^{\circ}\text{C}$) Expansion (cm^3)

0	0.13
2	0.03
4	0
6	0.03
10	0.12
12	0.48

Question 6

Calc Book Sec. 4.2

The table below gives the numbers of births on each day of a week in a maternity hospital.

Day	Number of births
Sunday	5
Monday	12
Tuesday	9
Wednesday	14
Thursday	9
Friday	17
Saturday	4

Suppose that births are equally likely to take place on each day of the week.

- (a) How would you use random digits to simulate the number of births on each day of a week in which there are 70 births?
- (b) Use your calculator to generate random digits for use in such a simulation. Make a tally of your results.
- (c) Do you think there is any evidence that more births occur on some days than on others.

Question 7

Unit 4 Sec. 4.1

A fair coin is one with an equal chance of coming down heads or tails (abbreviated to H and T respectively) when tossed. This means that a fair coin has an equal chance of landing either HH, HT, TH, TT when pairs of tosses are recorded. A computer-simulated fair coin is tossed 100 times and the 50 pairs of tosses are recorded as one of the four possibilities. The results of ten such simulations are summarized as follows:

Outcome	Simulation										Own
	1	2	3	4	5	6	7	8	9	10	
HH	8	7	18	6	14	12	12	14	14	9	
HT	17	20	9	12	15	14	16	10	11	13	
TH	12	13	13	18	11	12	11	13	8	11	
TT	13	10	10	14	10	12	11	13	17	17	

Perform the experiment yourself, by tossing a coin 50 pairs of times, and record the result in the last column of the table. The question you are to investigate is: Is your coin fair?

- (a) For each of the ten computer simulations, calculate the ratio of the maximum frequency to the minimum frequency.
- (b) Draw a boxplot of the ratios obtained in part (a) in order to see, pictorially, the variation due to chance.
- (c) Calculate the ratio of the maximum frequency to the minimum frequency for your coin and, using the boxplot, comment on the result.

Question 8

Unit 4 Sec. 4.1 and Calc Book Sec. 4.3

Find the standard deviation and the relative spread for each of the batches of data in Question 4 for Unit 2 (on page 9 of this resource book).

Question 9**Unit 4 Sec. 4**

The table below gives the pulse rate at rest of each of 30 students (19 men and 11 women) at a large college.

Men	58	60	62	62	64	66	66	66	68	68	70	70	72	72	74	76
	84	90	92													
Women	62	62	68	78	78	80	82	88	96	96	100					

One question of interest is whether the pulse rates of men and women differ. A secondary question is whether the amount of variation in pulse rates is similar among men and among women.

- Investigate these questions by preparing boxplots using a common axis. What do you conclude from your boxplots?
- Calculate the standard deviation of each batch of data. Comment briefly on your results.

Question 10**Unit 4 Sec. 4**

The table below gives the plasma beta endorphin concentrations of 11 runners after they had completed a Tyneside Great North Run and of another 11 runners after they had collapsed from exhaustion near the end of the race.

Finishers	29.6	25.1	15.5	29.6	24.1	37.8	20.2	21.9	14.2	
	34.6	46.2								
Collapsed runners	66	79	123	169	110	72	414	162	84	
	102	144								

Plasma beta endorphin is a constituent of human blood. Its concentration is measured in pmol/l, that is in picomoles (10^{-12} moles) per litre.

Clearly the plasma beta endorphin concentrations are higher for the collapsed runners than for those who finished the race, but it is less clear whether the concentrations are more or less variable for the collapsed runners.

- What are the advantages and disadvantages of using each of the following measures of variation in this situation?
 - Range
 - Interquartile range
 - Standard deviation
 - Relative spread
- Which measure do you think is the most appropriate to use here and why?
- For each set of runners, calculate the measure you chose in part (b) and comment briefly on your results.

Question 11

Unit 4 Sec. 4

For each of the following batches of data (from Question 1 for *Unit 3* on page 16 of this resource book), calculate the spread of the data using the four measures: range, interquartile range, standard deviation and relative spread. Comment on whether any of the measures of spread is not appropriate in any of the four cases.

- (a) 1 2 6 7 5 4 1 8
- (b) 1 2 3 2 1 3 1 1
- (c) 9 5 4 1 99 4 4 1
- (d) 1 30 20 55 3 78 21 63

Unit 5 Seabirds

Question 1

Unit 5 Sec. 1.3

Classify the following investigations according to which type you think they are: summarizing, comparing or looking for relationships.

- (a) Are women's pulse rates greater than men's?
- (b) Does ice-cream consumption vary with temperature?
- (c) Do people live longer than they did in the nineteenth century?
- (d) What is the average wage for men working full-time?
- (e) Do smokers have more time off for illness than non-smokers?
- (f) How long does a typical battery last in a torch?
- (g) Is there a link between TMA scores and examination marks?
- (h) Are the prices charged by supermarkets lower than those charged by corner shops?

Question 2

Unit 5 Sec. 2.3

Jim Poole recorded the number of apparently occupied fulmar nests on six areas of the Skomer cliffs on seven dates during June 1993. The following data were collected.

Date in June	7	13	17	22	24	27	30
North Haven	17	17	19	20	17	18	16
South Haven (east)	20	19	20	23	19	20	18
South Haven (west)	33	37	37	29	32	32	27
High Cliff	38	36	34	31	29	29	29
Bull Hole	22	22	21	18	20	19	17
The Wick	70	72	67	60	61	59	57
Totals	200	203	198	181	178	177	164

According to Jim, the decrease from 200 to 164 may be linked to non-breeding birds leaving the colonies or to birds losing eggs and therefore no longer being permanent residents.

Calculate the median, the interquartile range and the relative spread of:

- (a) the total number of fulmar nests;
- (b) the nests at North Haven;
- (c) the nests at the Wick.

Comment on your results.

Question 3

Unit 5 Sec. 2

An assistant warden on Skomer counted the number of razorbills at Bull Hole by dividing the cliff face into eight sections and counting birds in each section five times. Here are his data.

Section	Counts				
A	18	18	16	15	16
B	34	32	31	33	33
C	47	45	45	47	45
D	7	7	7	7	8
E	22	23	20	21	21
F	25	37	27	23	25
G	32	34	32	31	31
H	35	34	46	36	34

How many razorbills should be recorded as being at Bull Hole?

Question 4

Unit 5 Sec. 2

The table below gives the estimated number of breeding pairs of three species of gulls on Skomer since 1962.

Year	Lesser black-backed gull	Herring gull	Greater black-backed gull
1962	1400	1070	260
1966	3150	1650	210
1970	3657	1904	160
1974	7042	2359	152
1979	9616	2941	104
1984	12249	645	25
1988	17309	479	30
1992	15286	525	40
1996	14400	401	58
1999	12028	374	65

- (a) Use the data from the table to draw three scatterplots which show the variation of the number of breeding pairs of each of the three species of gulls over the years 1962 to 1999. Plot years on the horizontal axis and numbers of breeding pairs on the vertical axis. Write a sentence or two to describe the way in which the numbers of breeding pairs of the three species of gulls have varied over the years.
- (b) Choosing suitable axes, draw scatterplots which show how the number of breeding pairs of the lesser black-backed gulls varies with the number of breeding pairs of each of the other two species. Comment on your scatterplots.

Question 5

Unit 5 Sec. 3.1

Which of the following are examples of continuous variables and which are discrete?

- (a) Times between successive earthquakes at a particular location.
- (b) Speeds of cars past a service station.
- (c) Numbers of children in families.
- (d) The yield of tomatoes on a plant in kilograms.

- (e) The number of apples on a tree.
- (f) The weight of apples harvested from a tree.
- (g) Heights of eleven-year-old girls.
- (h) The numbers of letters delivered to a house each day.
- (i) Times taken by an individual to travel to work each day.

Question 6**Unit 5 Sec. 5**

A manufacturer claims that its new brand of light bulbs last on average 10 000 hours. You are asked by the trade association to investigate this claim.

- (a) Classify the problem as one of summarizing, comparing or looking for relationships.
- (b) Pose a precise question to investigate how long this brand of light bulb lasts.
- (c) The times that each of 1000 light bulbs lasted are summarized below.

Time range (hours)	Frequency
7 000–8 000	2
8 000–9 000	39
9 000–10 000	163
10 000–11 000	338
11 000–12 000	350
12 000–13 000	99
13 000–14 000	9

Choose an appropriate method to analyse the data (mean, median and interquartile range, boxplot or scatterplot) and carry out your analysis.

- (d) Interpret your results.

Hint: use the central value of each range to represent each range (for example, 7500 for the range 7000–8000).

Question 7**Unit 5 Sec. 5**

A motoring organization has performed a survey to investigate trends in the motoring habits of its members. One part of the data from the survey is shown in the table below. What the organization wants to know from these data is how many cars there are in a typical member's household.

Number of cars owned	Number of respondents
1	59
2	31
3	10

- (a) Classify the problem as one of summarizing, comparing or looking for relationships.
- (b) Pose a precise question for the motoring organization.
- (c) Choose an appropriate method to analyse the data (mean, median and interquartile range, boxplot or scatterplot) and carry out your analysis.
- (d) Interpret your results.

Question 8

Unit 5 Sec. 5

An advertisement claims that Brand X batteries last three times longer than ‘ordinary’ batteries. You are asked to investigate this claim.

- (a) Classify the problem as one of summarizing, comparing or looking for relationships.
- (b) Pose a precise question to investigate the claim in the advertisement.
- (c) The following are the data that the manufacturer provided to support the claim in the advertisement.

Duration of ordinary battery	Frequency	Duration of Brand X battery	Frequency
2000–2100	7	5800–5900	2
2100–2200	21	5900–6000	15
2200–2300	34	6000–6100	47
2300–2400	26	6100–6200	34
2400–2500	11	6200–6300	2
2500–2600	1		

Choose an appropriate method to analyse the data (mean, median and interquartile range, boxplots or scatterplot).

- (d) Interpret your results.

Question 9

Unit 5 Sec. 5

A mathematics examination paper contains two optional questions. Suppose that, after the exam, you were asked by the chief examiner to check whether the two options were of equal difficulty. You may assume that the number of people taking the exam was large enough so that the differences in marks can be attributed to the questions rather than the people choosing the questions. The data are given in the frequency table below.

Marks out of 10	Question 1	Question 2
0	1	2
1	5	1
2	14	8
3	18	12
4	23	16
5	27	28
6	16	22
7	10	21
8	5	10
9	5	3
10	11	5

- (a) Classify the problem as one of summarizing, comparing or looking for relationships.
- (b) Pose a precise question to investigate the chief examiner’s question.
- (c) Choose an appropriate method to analyse the data (mean, median and interquartile range, boxplots or scatterplot).
- (d) Interpret your results.

Question 10

Unit 5 Sec. 5

A fanatical sports club wishes to investigate the old adage ‘a healthy body makes a healthy mind’. They have available the following information about eight schools.

School	A	B	C	D	E	F	G	H
Area of playing fields (acres)	22	9	13	35	15	20	30	38
Number of exam passes in mathematics	48	22	49	70	30	74	77	50

- (a) Classify the problem as one of summarizing, comparing or looking for relationships.
- (b) Frame a precise question which you could investigate using these data.
- (c) Choose an appropriate method to analyse the data (mean, median and interquartile range, boxplots or scatterplot).
- (d) Interpret your results.

Solutions

Unit 1

1 For this question it is sensible to work in pounds and set your calculator to display answers to two decimal places. Alternatively you can do the appropriate rounding yourself.

- (a) £1020.00
- (b) £425.00
- (c) £8.13
- (d) £16.79
- (e) £16 458.75
- (f) £4544.80
- (g) £1.32
- (h) 0. No need to use the calculator here. A 100% decrease of any amount produces 0.

2 This question contains a mixture of pounds and pence but in *each part* the units are consistent. To enter the data on the calculator you can either use the actual units given in each part (as shown below), or if you prefer, convert everything to pounds. However, the calculated answers are percentages and in this case it is sufficiently accurate to quote these to the nearest whole number. You can either set your calculator to display zero decimal places, or do the rounding yourself.

- (a) Pint of milk

$$\frac{\text{increase in price}}{\text{original price}} \times 100\% = \frac{8}{36} \times 100\% \simeq 22\%$$

or

$$\frac{\text{new price}}{\text{original price}} \times 100\% = \frac{44}{36} \times 100\% \simeq 122\%$$

So either way the percentage price increase is approximately 22%.

- (b) Small car

$$\frac{\text{increase in price}}{\text{original price}} \times 100\% = \frac{2335}{7990} \times 100\% \simeq 29\%$$

or

$$\frac{\text{new price}}{\text{original price}} \times 100\% = \frac{10325}{7990} \times 100\% \simeq 129\%$$

So either way the percentage price increase is approximately 29%.

- (c) 10-minute phone call

$$\frac{\text{increase in price}}{\text{original price}} \times 100\% = \frac{-5}{60} \times 100\% \simeq -8\%$$

or

$$\frac{\text{new price}}{\text{original price}} \times 100\% = \frac{55}{60} \times 100 \simeq 92\%$$

So either way the percentage price *decreased* by approximately 8%.

- (d) A basic radio

$$\frac{\text{increase in price}}{\text{original price}} \times 100\% = \frac{2.80}{6.99} \times 100\% \simeq 40\%$$

or

$$\frac{\text{new price}}{\text{original price}} \times 100\% = \frac{9.79}{6.99} \times 100\% \simeq 140\%$$

So either way the percentage price increase is approximately 40%.

The radio has shown the greatest percentage price rise since 1994.

3

- (a) The fraction is $\frac{20}{100} \times £30 = £6$.
- (b) You would inherit $\frac{15}{100} \times £1000 = £150$.
- (c) This year's harvest would be

$$\frac{150}{100} \times 50\,000 = 75\,000 \text{ bushels.}$$

- (d) The total cost of the computer is

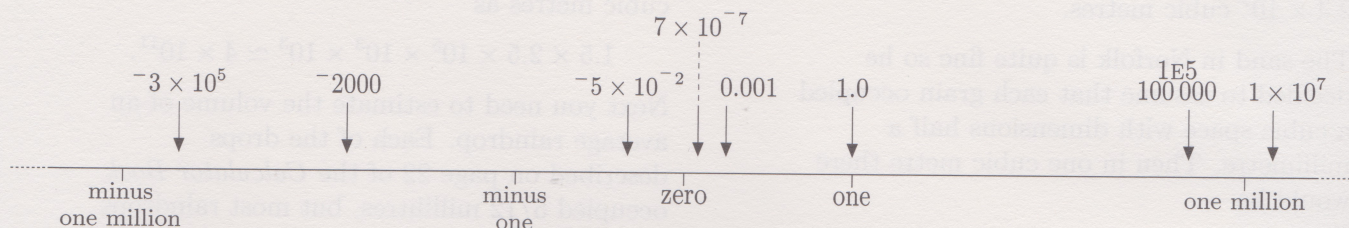
$$\frac{117.5}{100} \times £999 = £1173.83$$

(to the nearest penny).

4

- (a) The total cost of lottery tickets is £6, so each person pays $600/11 \simeq 55$ pence (to the nearest penny). There would be 5 pence left over.
- (b) Each person is entitled to $\frac{1}{11} \times 100\% = 9.1\%$ of the winnings (rounded to one decimal place).
- (c) Each person would win $£8\,000\,000 \div 11 = £727\,273$ (to the nearest pound).

5



6 Notice that a 'billion' is used in this question according to what is now the standard definition—a billion is a thousand million or 10^9 .

- (a) 6×10^9 is 6 billion so the increase from 5.4 billion is 0.6 billion or 0.6×10^9 . With 250 000 extra people per day, this increase will take $0.6 \times 10^9 \div 250\,000$ days. Dividing the answer by 365 gives just over six and a half years. So there would have been 6 billion people on the planet during 1998.

The whole calculation can be done at once on the calculator, as shown below (answers are rounded to three decimal places). Then the same entry can be used again in an edited form for the calculations in the next parts of the question.

- (b) (i) The *longest* time to reach 6 billion people uses the *smallest* possible initial population (5.35 billion) and the *smallest* possible increase 245 000. Repeating the calculation for part (a) using 5.35 billion and 245 000 gives the longest time to reach 6 billion people as nearly 7.3 years. To calculate the *shortest* time you need to use the *largest* initial population (5.45 billion) and the *largest* possible increase (255 000). This gives an answer of about 5.9 years.

Notice the large range of possible correct estimates. If the initial data were correct to two significant figures then the estimated times could be anywhere between 5.9 and 7.3 years!

- (ii) Here, since the daily increase is nearer 250 000 than 200 000, the smallest possible increase must have been 225 000. Repeating the calculation using 5.35 billion and 225 000 gives the longest time to reach 6 billion people as about 7.9 years.

- (c) According to UN 'best estimates', it seems that the population took about 7.8 years to reach 6 billion. This could mean that the 1992 estimates were rounded in the way suggested in part (b)(ii). However, look again at the wording in part (a) 'Assuming that the daily increase remains constant'—perhaps the daily increase fell, rather than remaining constant.

It is likely that the UN 'best estimates' quoted in part (c) used a more recent estimate of current population and daily growth than that quoted in part (a).

7

- (a) *Little grains of sand ...*

You need to define what is meant by 'the beach':

how long is it, i.e. how far along the coast does it extend?

how wide is it, i.e. how far out to sea does it extend?

how deep is the sand?

You need to decide whether the beach is comprised entirely of sand.

You need to decide what size a typical grain of sand might be (some sands are finer than others).

One member of the course team decided to think about a large expanse of sandy beach in north Norfolk. He decided that it was 6 km long, an average of 500 m wide and probably 10 m deep. He calculated the volume of the beach in cubic metres as

$$6000 \times 500 \times 10 = 3 \times 10^7.$$

He estimated that 80% of the beach was probably sand giving the volume of sand as 2.4×10^7 cubic metres.

The sand in Norfolk is quite fine so he decided to assume that each grain occupied a cubic space with dimensions half a millimetre. Then in one cubic metre there would be

$$\begin{aligned} &2000 \times 2000 \times 2000 \text{ grains} \\ &= 8 \times 10^9 \text{ grains.} \end{aligned}$$

So his estimate for the number of grains of sand on the beach was

$$\begin{aligned} &2.4 \times 10^7 \times 8 \times 10^9 = 19.2 \times 10^{16} \\ &\text{or roughly } 2 \times 10^{17}. \end{aligned}$$

(b) *Needles in a ...*

Here are some possible estimates for a particular forest where there are several large Norway Spruce plantations.

(i) How many needles on the average tree?

Average number of needles on a 'twig': 200
Average number of twigs on a main branch: 400
Average number of main branches per tree: 100

So needles on an average tree = 8×10^6 .

(ii) How many trees per hectare (a hectare is a square 100 m by 100 m)?

In the plantations, trees are growing about 3 m apart. So there are about $30 \times 30 = 9 \times 10^2$ trees in a 100 m by 100 m square.

(iii) How many hectares of Norway Spruce plantations are there in the forest?

From the Ordnance Survey map, the forest extends over about 35 square kilometres but probably only 20% of this is planted with Norway Spruce.

This is about 7 square kilometres or about 7×10^2 hectares (there are 100 hectares in 1 square kilometre).

$$\begin{aligned} &\text{Total number of needles in the forest} \\ &= 8 \times 10^6 \times 9 \times 10^2 \times 7 \times 10^2 \\ &= 504 \times 10^{10} \\ &\simeq 5 \times 10^{12}. \end{aligned}$$

(c) *Raindrops keep falling ...*

One way of proceeding would be to find out the average rainfall for the UK each year (at a guess this might be 1.5 m) and to multiply

this by the area of the UK (250 000 square km). This gives the volume of rainfall in cubic metres as

$$1.5 \times 2.5 \times 10^5 \times 10^3 \times 10^3 \simeq 4 \times 10^{11}.$$

Next you need to estimate the volume of an average raindrop. Each of the drops described on page 22 of the *Calculator Book* occupied 5/12 millilitres, but most raindrops are much smaller than this. Perhaps 0.1 millilitres would be a reasonable estimate. Then $\frac{1000}{0.1} = 10\,000$ raindrops would be needed to produce a litre of rainwater, and for a cubic metre you would need $10\,000 \times 1000 = 10^7$ raindrops. So the number of raindrops falling on the UK each year may be about $4 \times 10^{11} \times 10^7 = 4 \times 10^{18}$.

- 8 (a) 2000.8523 (b) 2.1544 (c) 1.7783
(d) 1.5849 (e) 7.0000 (f) 7.0000
(g) 2.0000 (h) 6.0000 (i) 3.0000
(j) 3.0000 (k) 2.0000 (l) 2.0000

9

- (a) The radius of the circular orbit is $6368 + 850 = 7218$ km. Its circumference is $7218 \times 2\pi \simeq 45\,000$ km to the nearest 1000 km.
(b) The speed of the satellite is about $\frac{7218 \times 2\pi}{100} \times 60 \simeq 27\,000$ km per hour, to two significant figures.

10

- (a) $\frac{1}{8}$ (b) $\frac{6}{25}$ (c) $\frac{199}{200}$ (d) $\frac{111}{1000}$
(e) $\frac{7}{40}$ (f) $\frac{32}{125}$ (g) $\frac{1}{250}$ (h) $\frac{5}{4} = 1\frac{1}{4}$

11

- (a) You should see the following sequence of numbers.

9 13 17 21 25 29 33 ...

Each time **ENTER** is pressed subsequently, 4 is added to the previous number.

- (b) A: 20 18 16 14 12 10 ...

- (i) Each number is obtained from the previous number by subtracting 2.
(ii) Calculator sequence:

20 **ENTER**
= 2 **ENTER** **ENTER** **ENTER** ...

B: 5 15 45 135 405 1215 ...

- (i) Each number is obtained from the previous number by multiplying by 3.
 (ii) Calculator sequence:

5 [ENTER]
 [×] 3 [ENTER] [ENTER] [ENTER] ...

C: 10 .1 10 .1 10 .1 ...

- (i) Each number is obtained from the previous number by calculating its reciprocal.

- (ii) Calculator sequence:

10 [ENTER]
 [x^{-1}] [ENTER] [ENTER] [ENTER] ...

D: 1 3 7 15 31 63 ...

- (i) Each number is obtained from the previous number by multiplying by 2 and adding 1.

- (ii) Calculator sequence:

1 [ENTER]
 [×] 2 [+] 1
 [ENTER] [ENTER] [ENTER] ...

Unit 2

1 Everybody's list will be different. However, there are problems with commodities that have been introduced to Britain in the last four centuries (for example, coffee) or have been invented in that period (for example, vacuum cleaners), and also commodities that have become outdated (for example, quill pens).

You might have considered the price of clothing (for example, shirts), but these would also be affected by fashion trends so would not be such a good choice.

The price of a luxury item (for example, wine) may only have been affordable by a small section of the population and so would not give an indication of how prices as a whole are moving. The same might apply to some foodstuffs (for example, meat).

Basic foodstuffs, such as bread and cheese, are a good choice, as they have been staple foods for a long time. It is hard to think of alternatives, so bread must be a reasonable choice. (It is not, of course, a perfect choice since many people in the countryside would have baked their own bread.)

2

- (a) The amount spent on bread as a percentage of the weekly food budget is

$$\frac{4.80}{65} \times 100 = 7.4\% \text{ (1 d.p.)}$$

- (b) The amount spent on milk as a percentage of the weekly food budget is

$$\frac{7.60}{65} \times 100 = 11.7\% \text{ (1 d.p.)}$$

- (c) The amount spent on fresh fruit and vegetables as a percentage of the weekly food budget is

$$\frac{15}{65} \times 100 = 23.1\% \text{ (1 d.p.)}$$

3

- (a) Percentage increase = $\frac{11}{131} \times 100\% = 8.4\%$ (1 d.p.).

- (b) Percentage increase = $\frac{10}{67} \times 100\% = 14.9\%$ (1 d.p.).

- (c) Percentage decrease = $\frac{9}{95} \times 100\% = 9.5\%$ (1 d.p.).

- (d) Percentage increase = 0%.

- (e) Percentage increase = $\frac{7}{59} \times 100\% = 11.9\%$ (1 d.p.).

- (f) Percentage decrease = $\frac{8}{326} \times 100\% = 2.5\%$ (1 d.p.).

- (g) Percentage decrease = $\frac{9}{150} \times 100\% = 6\%$.

4

- (a) The mean is

$$\frac{63}{8} = 7.875.$$

Sorting the eight numbers into ascending order produces the following:

4 6 7 7 8 9 10 12

The median is the mean of the two middle numbers, 7 and 8:

$$\text{median} = \frac{7+8}{2} = 7.5.$$

- (b) The mean is

$$\frac{120}{5} = 24.$$

Writing the numbers in ascending order gives the following.

21 23 24 26 26

The median is the middle number, 24.

- (c) The mean is

$$\frac{615}{6} = 102.5.$$

Sorting the numbers into ascending order gives the following.

92 98 101 102 107 115

The median is the mean of the two middle numbers:

$$\text{median} = \frac{101 + 102}{2} = 101.5.$$

(d) The mean is

$$\frac{444}{7} \simeq 63.4.$$

Sorting the numbers into ascending order gives the following.

38 47 57 64 71 74 93

The median is the middle number, 64.

5 Let x be the number of siblings and f the frequency. The mean number of brothers and sisters is

$$\frac{\sum xf}{\sum f} = \frac{0 \times 7 + 1 \times 18 + 2 \times 5 + 3 \times 2 + 4 \times 0 + 5 \times 1}{7 + 18 + 5 + 2 + 0 + 1} = \frac{39}{33} \simeq 1.2.$$

6 The prices per litre are the values (x), and the amounts bought are the weights (w). The mean price per litre paid by the motorist was

$$\frac{\sum xw}{\sum w} = \frac{7565.51}{88.5} \simeq 85.5 \text{ pence.}$$

7 Weighted mean = $\frac{9 \times 2 + 8 \times 1}{2 + 1} = 8\frac{2}{3}$ kg.

8 If r represents the price ratios and w represents the weights then the basket price ratio is $\sum rw / \sum w$.

(a) The basket price ratio for January 2003 relative to January 2002 is

$$\frac{\sum rw}{\sum w} = \frac{782.365}{770} \simeq 1.016.$$

(b) The basket price ratio for January 2004 relative to January 2003 is

$$\frac{\sum rw}{\sum w} = \frac{796.545}{770} \simeq 1.034.$$

9

(a) Between January 1997 and January 2004 the average price of a pint of milk rose by

$$\frac{1}{36} \times 100 \simeq 2.8\%.$$

(b) Between January 1998 and January 2002 the average price of a pint of milk rose by

$$\frac{2}{35} \times 100\% \simeq 5.7\%.$$

(c) The price ratios relative to January 1997 are given in the table below (to 3 d.p.).

Year	1998	1999	2000	2001
Price ratio	0.972	0.944	0.944	0.972
Year	2002	2003	2004	
Price ratio	1.028	1.028	1.028	

(d) The values of the price index calculated using the base-year method are calculated by multiplying the price ratios in part (c) by 100. The values are given in the table below.

Year	1997	1998	1999	2000
Index	100.0	97.2	94.4	94.4
Year	2001	2002	2003	2004
Index	97.2	102.8	102.8	102.8

(e) (i) The price ratio for 2002 relative to 1998 is

$$\frac{102.8}{97.2} = 1.058.$$

Between January 1998 and January 2002 the average price of a pint of milk rose by

$$(1.058 - 1) \times 100\% \simeq 6\%.$$

(Note that this calculation would give a percentage increase of 5.8% to 1 d.p. The calculation in part (b) gave a percentage increase of 5.7% to 1 d.p. for the same thing. The values differ because here the rounded price index has been used.)

(ii) The price ratio for 2000 relative to 1998 is

$$\frac{94.4}{97.2} = 0.971.$$

This is less than one because, between January 1998 and January 2000, the average price of a pint of milk *decreased*. The percentage change = $(0.971 - 1) \times 100\% \simeq -2.9\%$, or a decrease of nearly 3%.

(f) The one-year price ratios (that is, the price ratios relative to the previous January) are given in the table below.

Year	1998	1999	2000	2001
One-year price ratio	0.972	0.971	1.000	1.029
Year	2002	2003	2004	
One-year price ratio	1.057	1.000	1.000	

The values of the price index calculated using the previous-year method are given in the following table.

The 1998 index is calculated by multiplying the 1997 index by 0.972, giving $100 \times 0.972 = 97.2$. The 1999 index is calculated by multiplying the 1998 index by

0.971, giving $97.2 \times 0.971 \simeq 94.4$. The other values are calculated by a similar method.

Year	1997	1998	1999	2000
Index	100.0	97.2	94.4	94.4
Year	2001	2002	2003	2004
Index	97.1	102.6	102.6	102.6

- (g) The base-year and previous-year values differ slightly for some years. The small discrepancies are due to rounding errors. If you used unrounded values throughout parts (c) and (d), then your results from the two methods should not be different.

10

- (a) Between January 1997 and January 2004 the average price of a dozen eggs rose by

$$\frac{10}{162} \times 100\% \simeq 6.2\%.$$

- (b) Between January 1998 and January 2002 the average price of a dozen eggs rose by

$$\frac{15}{157} \times 100\% \simeq 9.6\%.$$

- (c) The price ratios relative to January 1997 are given in the table below (to 2 d.p.).

Year	1998	1999	2000	2001
Price ratio	0.969	0.969	1.006	1.037
Year	2002	2003	2004	
Price ratio	1.062	1.049	1.062	

- (d) The values of the price index calculated using the base-year method are calculated by multiplying the price ratios in part (c) by 100. The values are given in the table below.

Year	1997	1998	1999	2000
Index	100.0	96.9	96.9	100.6
Year	2001	2002	2003	2004
Index	103.7	106.2	104.9	106.2

- (e) (i) The price ratio for 2002 relative to 1998 is

$$\frac{106.2}{96.9} \simeq 1.096.$$

Between January 1998 and January 2002 the average price of a dozen eggs rose by

$$(1.096 - 1) \times 100\% = 9.6\% \simeq 10\%.$$

- (ii) The price ratio for 2000 relative to 1998 is

$$\frac{100.6}{96.9} \simeq 1.038.$$

Between January 1998 and January 2000 the average price of a dozen eggs rose by

$$(1.038 - 1) \times 100\% \simeq 4\%.$$

- (f) The one-year price ratios are given in the table below.

Year	1998	1999	2000	2001
One-year price ratio	0.969	1.000	1.038	1.031
Year	2002	2003	2004	
One-year price ratio	1.024	0.988	1.012	

The values of the price index calculated using the previous-year method are given in the following table.

Year	1997	1998	1999	2000
Index	100.0	96.9	96.9	100.6
Year	2001	2002	2003	2004
Index	103.7	106.2	104.9	106.2

- (g) The base-year and previous-year values are the same to 3 d.p., despite rounding errors.

11 The method is described in detail in *Unit 2*, Section 5.2.

July 2004

$$\text{All-item price ratio} = \frac{\sum rw}{\sum w} = \frac{1020.241}{1000} = 1.020\ 241$$

$$\text{Value of RPI} = 183.1 \times 1.020\ 241 \simeq 186.8$$

September 2004

$$\text{All-item price ratio} = 1.027\ 204$$

$$\text{Value of RPI} = 183.1 \times 1.027\ 204 \simeq 188.1$$

October 2004

$$\text{All-item price ratio} = 1.029\ 886$$

$$\text{Value of RPI} = 183.1 \times 1.029\ 886 \simeq 188.6$$

12

- (a) If the average price of an item rises then the price ratio for that item is greater than 1, so the value of the RPI is likely to rise as a result. Since the price of cheese and the price of tobacco both rose between October 2003 and October 2004, each price change is likely to lead to an increase in the value of the RPI.
- (b) Both increases are likely to be small as cheese and tobacco form only a small part of an average household's expenditure: no single group, subgroup or section has a large effect on its own unless there is a very large change in its price, which is not the case here.

- (c) The weights are proportional to average UK household spending. You may know (or have guessed) that UK households spend more on tobacco than on cheese (on average). Thus the subgroup Tobacco has the larger weight (typically about 30 in the UK in the early years of this century). Cheese is only one out of a large number of sections in the Food subgroup and so has a small weight (the weight of the Food subgroup is typically about 115, and the 2004 weight for cheese is 3). (Don't worry if you got this wrong because you thought that an average household spent more on cheese than tobacco!)
- (d) The rise in the price of tobacco is likely to have the greater effect on the value of the RPI because tobacco has a larger weight than cheese and because the percentage increase in the price of tobacco was greater than that of cheese.

13

- (a) This exercise is similar to Exercise 1 in Frame 6 of *Unit 2*, Subsection 6.1.
- (i)
- $$\text{Price ratio} = \frac{\text{value of CPI in February 2003}}{\text{value of CPI in February 2002}} = \frac{109.0}{107.3} = 1.015\,843\,43.$$
- CPI in February 2003 $\simeq$ 101.6% of CPI in February 2002.
- The annual rate of inflation in February 2003 was 1.6%.
- (ii)
- $$\text{Price ratio} = \frac{\text{value of CPI in May 2003}}{\text{value of CPI in May 2002}} = \frac{109.7}{108.4} = 1.011\,992\,62.$$
- CPI in May 2003 = 101.2% of CPI in May 2002.
- The annual rate of inflation in May 2003 was 1.2%.
- (iii)
- $$\text{Price ratio} = \frac{\text{value of CPI in September 2003}}{\text{value of CPI in September 2002}} = \frac{110.2}{108.7} = 1.013\,799\,448.$$
- CPI in September 2003 = 101.4% of CPI in September 2002.

The annual rate of inflation in September 2003 was 1.4%.

(iv)

$$\begin{aligned} \text{Price ratio} &= \frac{\text{value of CPI in November 2003}}{\text{value of CPI in November 2002}} \\ &= \frac{110.3}{108.9} = 1.012\,855\,831. \end{aligned}$$

CPI in November 2003 = 101.3% of CPI in November 2002.

The annual rate of inflation in November 2003 was 1.3%.

- (b) This exercise is similar to Exercise 2 in Frame 9 of *Unit 2*, Subsection 6.1.
- (i) The value of the pension in April 2003 should be

$$\begin{aligned} \text{pension in April 2002} \times \frac{\text{RPI in April 2003}}{\text{RPI in April 2002}} \\ = £100 \times \frac{181.2}{175.7} \simeq £103.13. \end{aligned}$$

(ii) The value of the pension in June 2003 should be

$$\begin{aligned} \text{pension in June 2002} \times \frac{\text{RPI in June 2003}}{\text{RPI in June 2002}} \\ = £85 \times \frac{181.3}{176.2} \simeq £87.46. \end{aligned}$$

(iii) The value of the pension in October 2003 should be

$$\begin{aligned} \text{pension in October 2002} \times \frac{\text{RPI in October 2003}}{\text{RPI in October 2002}} \\ = £121 \times \frac{182.6}{177.9} \simeq £124.20. \end{aligned}$$

(iv) The value of the pension in September 2003 should be

$$\begin{aligned} \text{pension in May 2002} \times \frac{\text{RPI in September 2003}}{\text{RPI in May 2002}} \\ = £94 \times \frac{182.5}{176.2} \simeq £97.36. \end{aligned}$$

- (c) This exercise is similar to Exercise 3 in Frame 12 of *Unit 2*, Subsection 6.1.

(i) The purchasing power of the pound in March 2003 compared with one year earlier was

$$\begin{aligned} \frac{\text{value of RPI in March 2002}}{\text{value of RPI in March 2003}} \times 100\text{p} \\ = \frac{174.5}{179.9} \times 100\text{p} \simeq 97\text{p}. \end{aligned}$$

- (ii) The purchasing power of the pound in September 2003 compared with one year earlier was

$$\frac{\text{value of RPI in September 2002}}{\text{value of RPI in September 2003}} \times 100\text{p} \\ = \frac{177.6}{182.5} \times 100\text{p} \simeq 97\text{p}.$$

- (iii) The purchasing power of the pound in November 2003 compared with May 2002 was

$$\frac{\text{value of RPI in May 2002}}{\text{value of RPI in November 2003}} \times 100\text{p} \\ = \frac{176.2}{182.7} \times 100\text{p} \simeq 96\text{p}.$$

- (iv) The purchasing power of the pound in December 2003 compared with January 1987 was

$$\frac{\text{value of RPI in January 1987}}{\text{value of RPI in December 2003}} \times 100\text{p} \\ = \frac{100}{183.5} \times 100\text{p} \simeq 54\text{p}.$$

14

- (a) The ratio height/head circumference is less than 3 for Abbas (2.94) and Carol (2.88), so Abbas and Carol have a relatively large head circumference for their height.
- (b) The ratio height/head circumference is greater than 3 for Derek (3.16), so Derek has a relatively small head circumference for his height. (The ratio is 3.01 for Beryl—very close to the average.)
- (c) The average head circumference for people 171 cm tall is approximately $171/3 \text{ cm} = 57 \text{ cm}$.

15

- (a) A and E are similar. B and D are similar.
- (b) The ratios of corresponding sides are $50/10 = 5$ and $100/20 = 5$. So that the sides of triangle B are five times those of A. The ratio is 1:5.
- (c) The lengths of the sides in the second triangle are $10/3$ times those in the first. So X is of length $\frac{10}{3} \times 15 = 50 \text{ m}$.

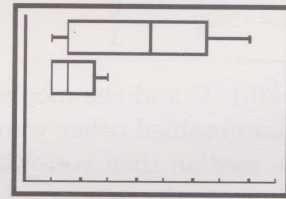
- 16 The picture is effectively a scale drawing of the person and house. From the picture, the house is four times larger than the person, which makes the house $4 \times 2 = 8$ metres high. (The ladder would have to be longer than this because it will be at an angle to the vertical.)

Unit 3

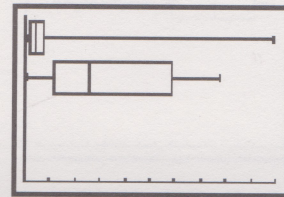
- 1 The mean and median for each batch of data is tabulated below.

Batch	Mean	Median
(a)	4.25	4.5
(b)	1.75	1.5
(c)	15.875	4
(d)	33.875	25.5

Boxplots for batches (a) and (b):



Boxplots for batches (c) and (d):



In the absence of other information about the data it is impossible to say which measure of the average is better for the batches of data. What you *can* say is that in batch (c) the mean is unduly affected by the one high value in the data and so the median is probably more appropriate.

In all of the other batches the case for the mean or the median is less clear cut and so either could be appropriate.

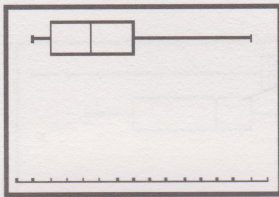
2

(a) The frequency table is:

Temperature (°C)	Frequency
16	3
17	6
18	3
19	3
20	2
21	4
22	4
23	0
24	0
25	3
26	1
27	0
28	0
29	1

(b) The mean is 20.1 °C and the median is 19.5 °C. (If you obtained other values for the mean and the median then you may have forgotten to include the number of times each temperature has occurred in your calculation—see Chapter 2 of the *Calculator Book* for details on how to do this.)

(c)



The boxplot is skewed. (In fact, it is right-skewed, since the right whisker is much longer than the left whisker.) So the median is more appropriate as a measure of the average temperature as it is less affected by the few hot days in the month.

3

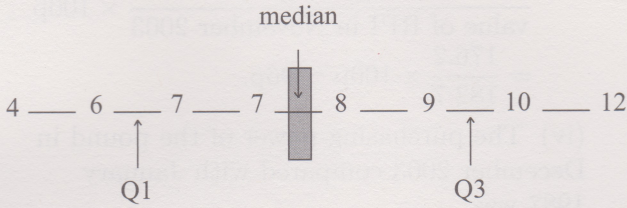
(a) The earnings ratios at the mean (correct to 1 d.p.) are as follows:

1998	$\frac{752}{906} \times 100\% \simeq 83.0\%$
1999	$\frac{799}{937} \times 100\% \simeq 85.3\%$
2000	$\frac{829}{970} \times 100\% \simeq 85.5\%$
2001	$\frac{859}{1013} \times 100\% \simeq 84.8\%$
2002	$\frac{916}{1044} \times 100\% \simeq 87.7\%$
2003	$\frac{952}{1091} \times 100\% \simeq 87.3\%$
2004	$\frac{1014}{1119} \times 100\% \simeq 90.6\%$

(b) Overall, these data show a fairly steady decrease in the gap between men's and women's earnings over the period 1998 to 2004 in Northern Ireland.

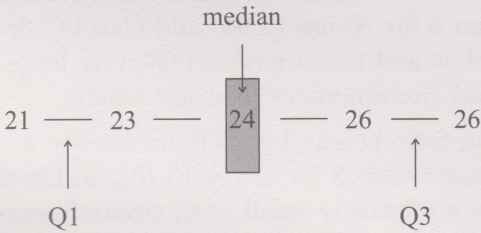
4 The numbers in each batch were written in ascending order in the solution to Question 4 for Unit 2. Diagrams below show where the median and quartiles are. In each case, the quartiles come half way along the batches of data formed by splitting the original batch at the median.

(a)



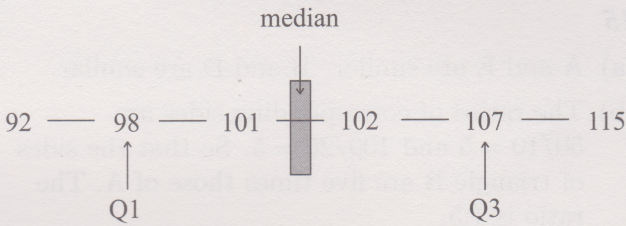
$min = 4, Q1 = 6.5, Q3 = 9.5, max = 12.$
 $range = max - min = 12 - 4 = 8.$
 $interquartile\ range = Q3 - Q1$
 $ = 9.5 - 6.5 = 3.$

(b)



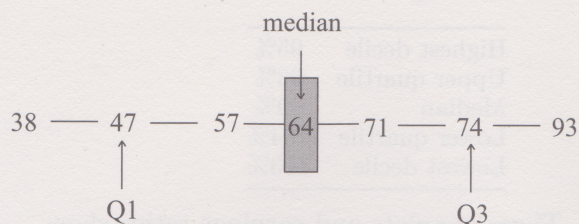
$min = 21, Q1 = 22, Q3 = 26, max = 26.$
 $range = 26 - 21 = 5.$
 $interquartile\ range = 26 - 22 = 4.$

(c)



$min = 92, Q1 = 98, Q3 = 107, max = 115.$
 $range = 115 - 92 = 23.$
 $interquartile\ range = 107 - 98 = 9.$

(d)



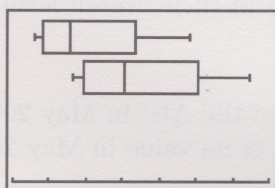
$$\min = 38, Q1 = 47, Q3 = 74, \max = 93.$$

$$\text{range} = 93 - 38 = 55.$$

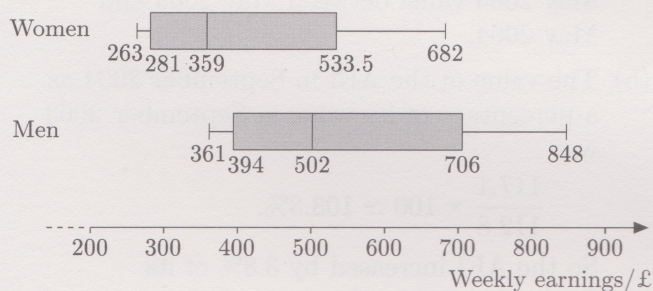
$$\text{interquartile range} = 74 - 47 = 27.$$

5

(a)



(b) Boxplots for the earnings of the sales representatives are shown below.



(c) From the boxplots it seems that the men's earnings are generally higher than the women's: all five values marked on a boxplot are higher for the men than for the women. The difference is greatest at the upper end of the earnings range.

(d) Both boxplots are right-skewed: in each case the right whisker is longer than the left whisker and the right-hand section of this box is longer than left-hand section. So the men's and women's earnings distributions are both right-skewed.

(e) For the women:

$$\text{range} = 682 - 263 = 419;$$

$$\text{interquartile range} = 533.5 - 281 = 252.5.$$

For the men:

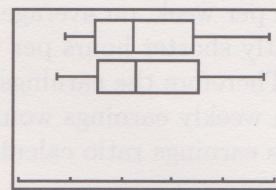
$$\text{range} = 848 - 361 = 487;$$

$$\text{interquartile range} = 706 - 394 = 312.$$

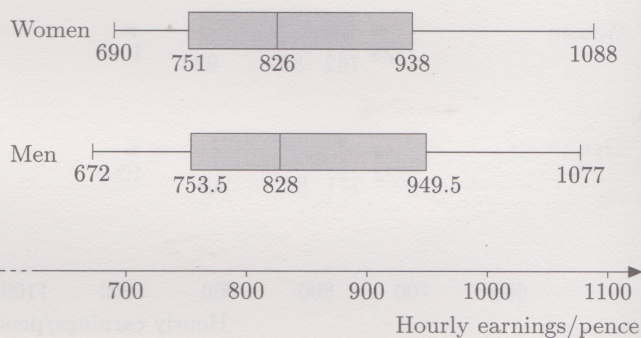
The spread is slightly greater for the men's earnings: both the range and the interquartile range are a bit greater for the men's earnings than for the women's earnings.

6

(a)



(b) Boxplots for the earnings of the postal workers, mail sorters, messengers and couriers are shown below.



(c) From the boxplots it seems that the men's earnings are generally very similar to the women's: all five values marked on a boxplot are similar for the men and women.

(d) Both boxplots are slightly right-skewed: for each boxplot, the right-hand whisker is longer than the left-hand whisker and the right-hand section of the box is longer than the left-hand section of the box. So both earnings distributions are slightly right-skewed.

(e) For the women:

$$\text{range} = 1088 - 690 = 398;$$

$$\text{interquartile range} = 938 - 751 = 187.$$

For the men:

$$\text{range} = 1077 - 672 = 405;$$

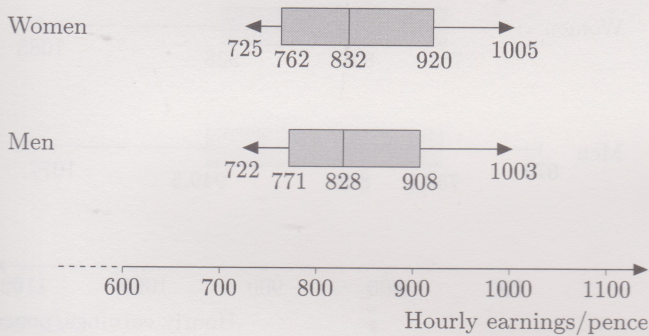
$$\text{interquartile range} = 949.5 - 753.5 = 196.$$

The spreads are very similar for men's earnings and women's earnings—though overall the spread for men's earnings is slightly larger. The range and interquartile range are both slightly larger for the men's earnings than for the women's.

7 Even if female workers in this group earned the same per hour as male workers, the females would earn less per week, on average, because they work slightly shorter hours per week than the males do. Therefore the earnings ratio calculated from weekly earnings would probably be less than the earnings ratio calculated from hourly earnings.

8

(a) The decile boxplots are shown below.



The decile boxplots show that the gross hourly earnings of the men are very similar to the gross hourly earnings of the women. The spread of earnings is also similar for male and female workers in this occupational group.

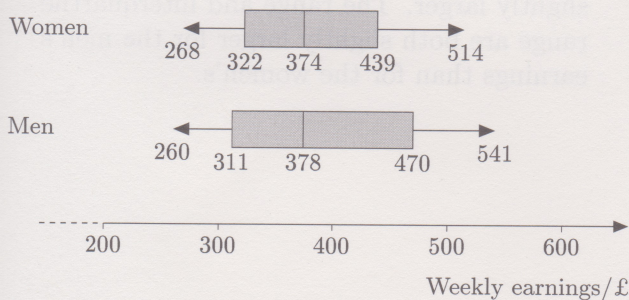
(b) The earnings ratios are as follows:

Highest decile	100%
Upper quartile	101%
Median	100%
Lower quartile	99%
Lowest decile	100%

These figures show that across the whole range of earnings, female postal workers, mail sorters, messengers and couriers earn about the same per hour as their male counterparts.

9

(a) The decile boxplots are shown below.



(b) The earnings ratios are as follows.

Highest decile	95%
Upper quartile	93%
Median	99%
Lower quartile	104%
Lowest decile	103%

(c) These boxplots and earnings ratios show that, for higher earnings levels, male youth and community workers earn a bit more than their female counterparts, but for lower earnings levels, it is the other way round. The main difference here between men and women is in the spread of weekly earnings rather than in their overall level.

10

(a) The value of the AEI in May 2004 as a percentage of its value in May 2003 was

$$\frac{115.3}{111.3} \times 100 \simeq 103.6\%.$$

So the AEI increased by 3.6% of its May 2003 value between May 2003 and May 2004.

(b) The value of the AEI in September 2004 as a percentage of its value in September 2003 was

$$\frac{117.1}{112.8} \times 100 \simeq 103.8\%.$$

So the AEI increased by 3.8% of its September 2003 value between September 2003 and September 2004.

11

(a) The real earnings for January 2004 compared with one year earlier were

$$\begin{aligned} & \frac{\text{AEI for Jan 2004}}{\text{AEI for Jan 2003}} \div \frac{\text{RPI for Jan 2004}}{\text{RPI for Jan 2003}} \times 100\% \\ &= \frac{118.3}{109.9} \div \frac{183.1}{178.4} \times 100\% \\ &= \frac{118.3}{109.9} \times \frac{178.4}{183.1} \times 100\% \simeq 104.9\%. \end{aligned}$$

(b) The real earnings for April 2004 compared with one year earlier were

$$\begin{aligned} & \frac{\text{AEI for Apr 2004}}{\text{AEI for Apr 2003}} \div \frac{\text{RPI for Apr 2004}}{\text{RPI for Apr 2003}} \times 100\% \\ &= \frac{115.6}{110.7} \div \frac{185.7}{181.2} \times 100\% \\ &= \frac{115.6}{110.7} \times \frac{181.2}{185.7} \times 100\% \simeq 101.9\%. \end{aligned}$$

- (c) The real earnings for June 2004 compared with one year earlier were

$$\frac{\text{AEI for Jun 2004}}{\text{AEI for Jun 2003}} \div \frac{\text{RPI for Jun 2004}}{\text{RPI for Jun 2003}} \times 100\%$$

$$= \frac{116.1}{111.5} \div \frac{186.8}{181.3} \times 100\%$$

$$= \frac{116.1}{111.5} \times \frac{181.3}{186.8} \times 100\% \simeq 101.1\%.$$

- (d) The real earnings for August 2004 compared with one year earlier were

$$\frac{\text{AEI for Aug 2004}}{\text{AEI for Aug 2003}} \div \frac{\text{RPI for Aug 2004}}{\text{RPI for Aug 2003}} \times 100\%$$

$$= \frac{116.8}{112.2} \div \frac{187.4}{181.6} \times 100\%$$

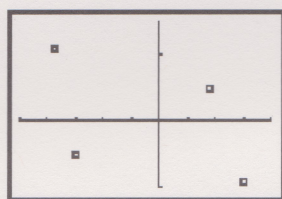
$$= \frac{116.8}{112.2} \times \frac{181.6}{187.4} \times 100\% \simeq 100.9\%.$$

Unit 4

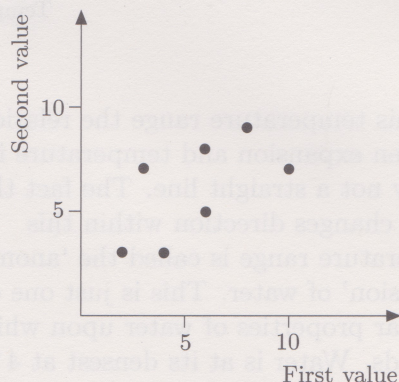
1

- (a) Note the wording of this question. 'On your calculator, produce a scatterplot of ...'. You were not expected to draw it on paper.

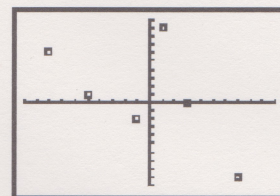
Depending on the window settings chosen you should get a scatterplot similar to the following.



(b)

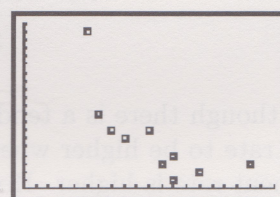


- (c) Axes marked from -10 to 10 are appropriate here (i.e. the Standard settings of the calculator).



2

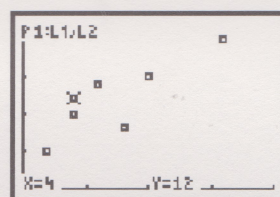
- (a) If you choose window settings running from 0 to 20 you will get a scatterplot similar to the following.



- (b) The scatterplot indicates, roughly speaking, that the more goals a team scores, the fewer goals are scored against them. This would seem reasonable since a 'good' team would tend to score more goals and concede fewer.

3

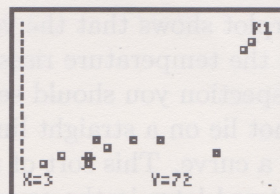
- (a) You are asked to draw a scatterplot. Using the calculator's TRACE facility is very helpful.



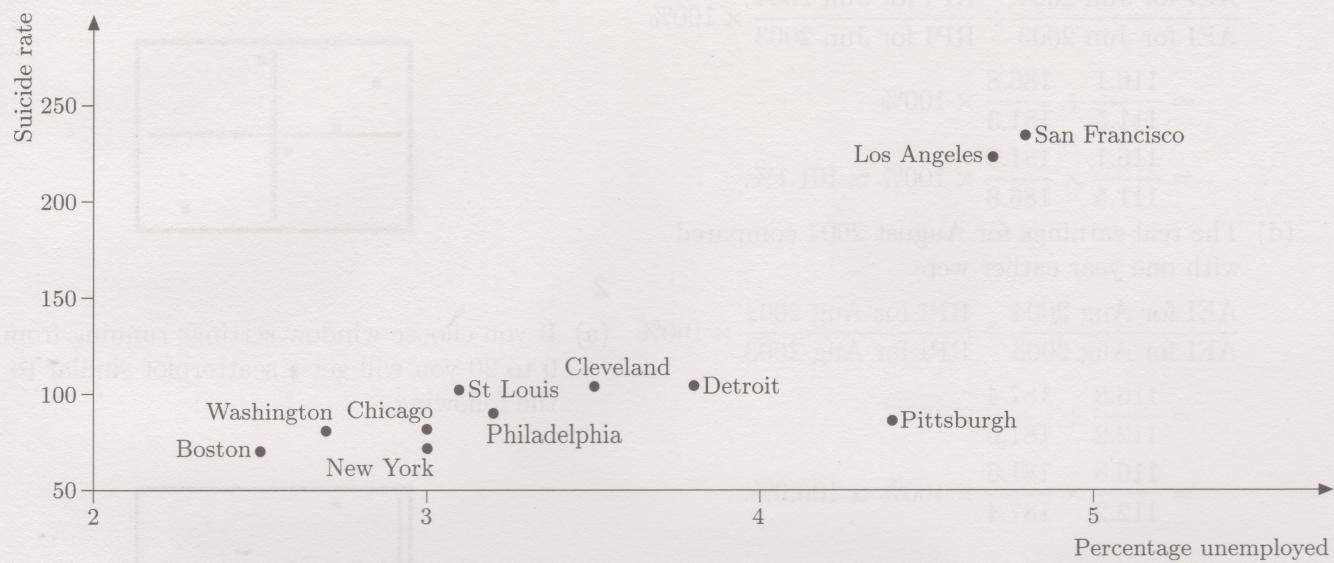
- (b) The data suggest that the number of accidents per 1000 employees is related to the age of the factory. However, the data set is very small; this pattern might not be apparent if more data were available.

4

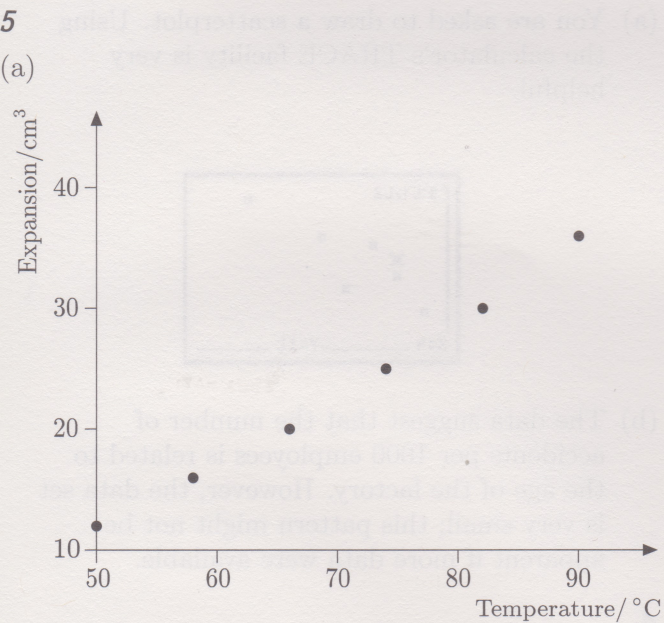
- (a)



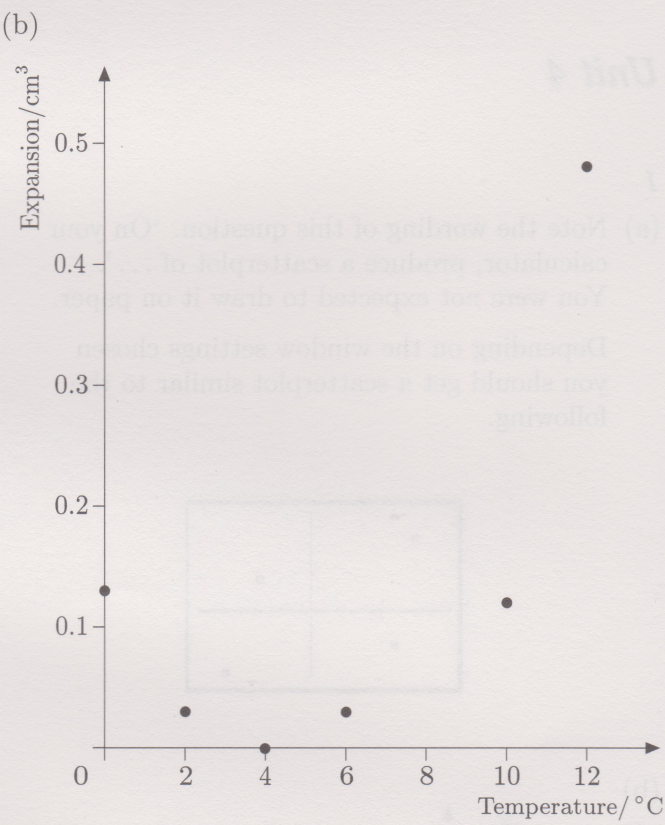
(b) A scatterplot is shown below.



(c) It looks as though there is a tendency for the suicide rate to be higher where the unemployment rate is higher. However, notice that if the two cities with the highest unemployment rates and suicide rates are omitted (Los Angeles and San Francisco), then the relationship between unemployment rate and suicide rate for the other cities is less obvious.



The scatterplot shows that the volume expands as the temperature rises. However, on close inspection you should see that the points do not lie on a straight line; rather, they lie on a curve. This sort of relationship will be explored later in the course.



For this temperature range the relationship between expansion and temperature is clearly not a straight line. The fact that the graph changes direction within this temperature range is called the 'anomalous expansion' of water. This is just one of the peculiar properties of water upon which life depends. Water is at its densest at 4°C, so the bottom of cold oceans is at a temperature of 4°C.

6

- (a) One possibility is to label the days of the week (Sunday to Saturday) 1 to 7. Then you could take 70 random digits between 1 and 7 and record a Sunday birth for each 1, a Monday birth for each 2, and so on.

An easier method would be to use the calculator's randInt command to produce the digits 1 to 7. You could produce a list of 70 such digits and store them in a list. Sorting the list would make it easier to work out the frequency of each digit. In Chapter 5 of the *Calculator Book* you will learn how to draw a frequency diagram (or histogram) which allows you to find the frequencies using the Trace facility.

(b)

```
randInt(1,7,70)→
L1
5 5 1 4 6 1 7 ...
```

A possible tally chart is shown below, but you will almost certainly have obtained different digits.

Day	Digit	Tally	Frequency
Sunday	1	1	11
Monday	2		10
Tuesday	3		9
Wednesday	4	11	12
Thursday	5		8
Friday	6		8
Saturday	7	11	12

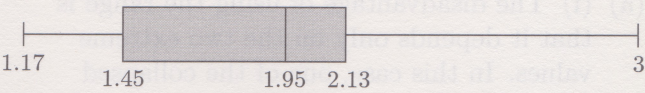
- (c) The actual numbers of births vary from day to day rather more than the simulated numbers of births. In particular, there were far fewer births at the weekend than on weekdays. Assuming the simulation is typical of what is likely to occur by chance, then it seems unlikely that the variation in the table in the question is due to chance alone. (Could it be that induced births, Caesareans, etc. are done on weekdays, so that there are fewer births in the hospital at weekends?)

7

- (a) The ratios for the ten simulations, rounded to two decimal places where relevant, are as follows.

Simulation	1	2	3	4	5
Ratio	2.13	2.86	2	3	1.5
Simulation	6	7	8	9	10
Ratio	1.17	1.45	1.4	2.13	1.89

- (b) A boxplot of the ratios is shown below.



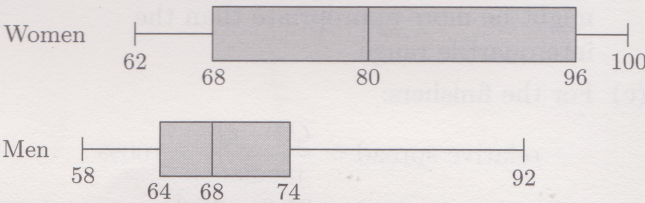
- (c) Only you can decide whether your coin is fair or biased. But remember, even if your ratio is outside the range of values covered by the boxplot, it could still have arisen purely by chance.

8 The standard deviations given below were obtained using the course calculator. The median and interquartile range used to calculate the relative spread in each case were obtained from the solutions to Question 4 for Unit 2 and Question 4 for Unit 3.

- (a) Standard deviation $\simeq 2.3$
Relative spread = $\frac{3}{7.5} \times 100\% = 40\%$
(b) Standard deviation $\simeq 1.9$
Relative spread = $\frac{4}{24} \times 100\% \simeq 16.7\%$
(c) Standard deviation $\simeq 7.2$
Relative spread = $\frac{9}{101.5} \times 100\% \simeq 8.9\%$
(d) Standard deviation $\simeq 16.9$
Relative spread = $\frac{27}{64} \times 100\% \simeq 42.2\%$

9

- (a) The boxplots are shown below.



All five points on the women's boxplot are higher, indicating that the women's pulse rates are generally higher than the men's. The box of the women's boxplot is nearly three times as long as the box of the men's boxplot, indicating that the women's pulse rates are more widely spread (as measured by the interquartile range).

- (b) For the men:
standard deviation $\simeq 9.2$. (to 1 d.p.)
For the women:
standard deviation $\simeq 12.7$. (to 1 d.p.)

The pulse rates of the women are more widely spread than those of the men.

10

- (a) (i) The disadvantage of using the range is that it depends only on the two extreme values. In this case, one of the collapsed runners had a much higher plasma beta endorphin concentration (414 pmol/l) than any of the others.
- (ii) The advantage of using the interquartile range is that it is unaffected by unusually small and large values, such as the unusually large value in the second batch.
- (iii) The advantage of the standard deviation is that it takes all values into account. However, the standard deviation is strongly influenced by unusually large and unusually small values, and there is an unusually large value for one of the collapsed runners.
- (iv) The advantage of the relative spread is that it is unaffected by unusually large and unusually small values, such as the unusually large value in the second batch.

When comparing the spread of two batches, the relative spread also takes the size of the values into account.

(b) Since there is an unusually large value in one of the batches, the range and standard deviation are not good measures to use. The median value for the collapsed runners (110) is more than four times the median value for the finishers (25), so the relative spread might be more appropriate than the interquartile range.

- (c) For the finishers:

$$\begin{aligned} \text{relative spread} &= \frac{Q3 - Q1}{\text{median}} \times 100\% \\ &= \frac{34.6 - 20.2}{25.1} \times 100\% \\ &= 57\%. \end{aligned}$$

For the collapsed runners:

$$\begin{aligned} \text{relative spread} &= \frac{162 - 79}{110} \times 100\% \\ &\simeq 76\%. \end{aligned}$$

The relative spread is greater for the collapsed runners, indicating that there is greater variation in the plasma beta endorphin concentrations for the collapsed runners than for the finishers.

11

Batch	Range	Interquartile range	Standard deviation	Relative spread
(a)	7	5	2.54	111%
(b)	2	1.5	0.83	100%
(c)	98	4.5	31.51	113%
(d)	77	47.5	26.57	186%

In (c) the range and standard deviation are unduly affected by the one very large value. This means that the interquartile range or the relative spread might be better measures of the spread for batch (c).

Unit 5

1

- (a) Comparing
- (b) Looking for relationships
- (c) Comparing
- (d) Summarizing
- (e) Comparing
- (f) Summarizing
- (g) Looking for relationships
- (h) Comparing

2

	Median	Interquartile range	Relative spread
All nests	181	23	13%
North Haven	17	2	12%
The Wick	61	11	18%

There was far greater variation in the data obtained from the Wick than in the data from North Haven, perhaps indicating that fulmars are less likely to breed successfully at the Wick.

3 There are two suspicious-looking pieces of data which should probably be regarded as outliers—the 37 in section F and the 46 in section H.

One approach to determining the number of razorbills that should be recorded is to calculate the median for each of the eight sections and then to add these up. This is a sensible approach since the outliers have no effect on this calculation. This approach gives the following.

A	B	C	D	E	F	G	H	Total
16	33	45	7	21	25	32	35	214

An alternative approach might be to calculate totals for each of the five counts and then to find the median of the totals. However, the outliers mean that the second and third totals would be questionable. The five totals are as follows:

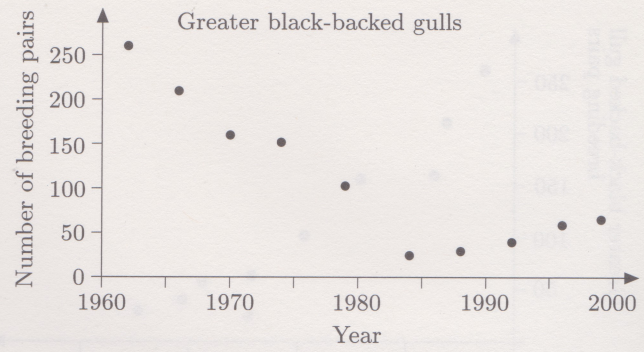
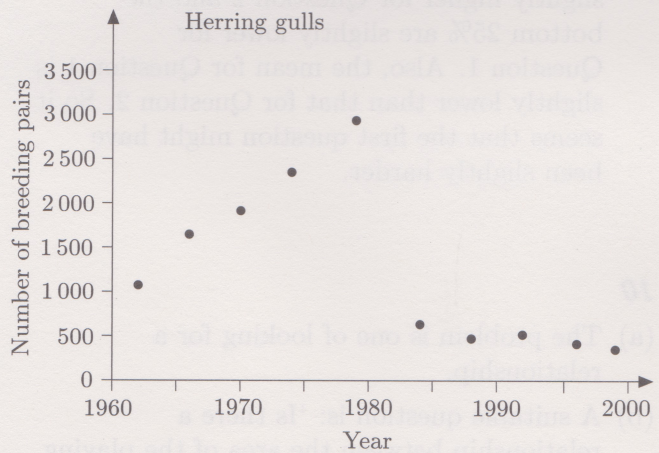
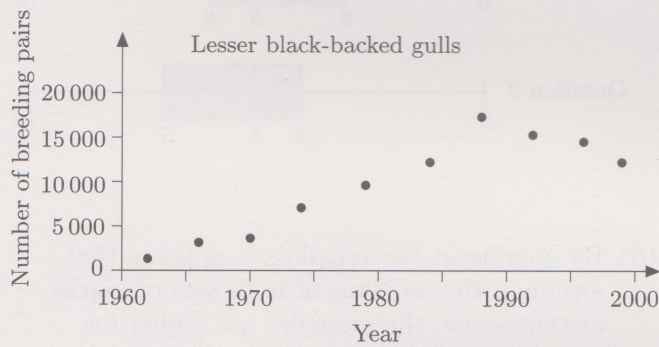
220 230 224 213 213

The median of all five totals is 220. It is probably more appropriate to discard the second and third totals, which contain the outliers, and to find the median of the remaining three, which is 213. However, it does not seem sensible to discard two-fifths of the data because of two questionable numbers, so the first approach is to be preferred.

So the number of razorbills at Bull Hole should be recorded as 214.

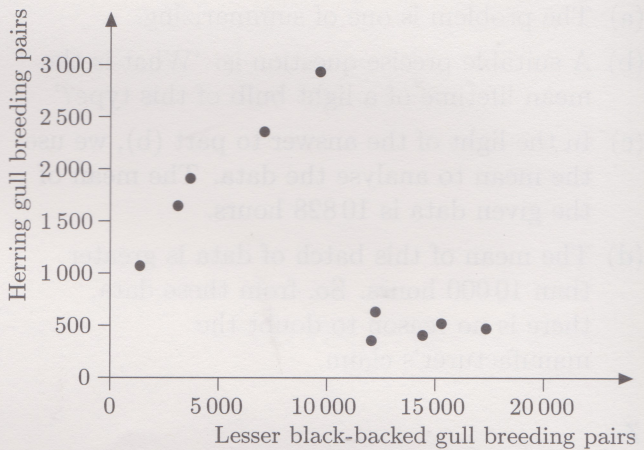
4

- (a) Scatterplots showing the variation in the numbers of breeding pairs of the three species of gulls are shown below.

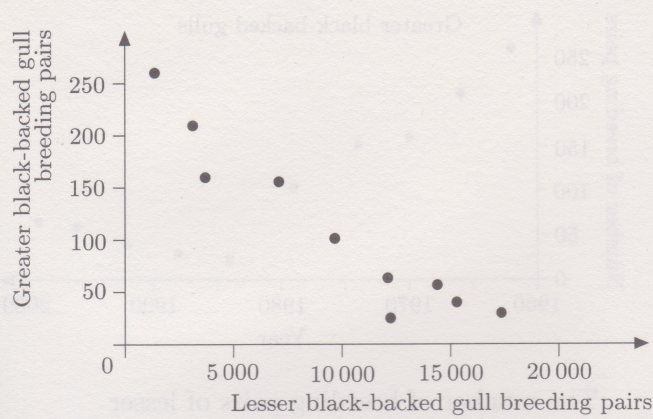


The number of breeding pairs of lesser black-backed gulls expanded rapidly until 1988 after which the numbers declined. The number of breeding pairs of herring gulls also increased rapidly until about 1980, after which it suffered severe reduction. (This was actually caused by an attack of botulism throughout the colony.) Since then the number has declined further. The number of breeding pairs of greater black-backed gulls steadily reduced until 1984, since when there has been a small gradual recovery.

(b)



There does not appear to be any overall relationship between the numbers of breeding pairs of the two species. However, the numbers of breeding pairs of the two species did both increase up to 1979.



There does appear to be a relationship between the numbers of breeding pairs of the two species. As one increases, the other decreases. However, this does not necessarily imply that an increase in one causes a decrease in the other, or vice versa.

- 5 Continuous: (a), (b), (d), (f), (g), (i).
Discrete: (c), (e), (h).

6

- (a) The problem is one of summarizing.
- (b) A suitable precise question is: ‘What is the mean lifetime of a light bulb of this type?’
- (c) In the light of the answer to part (b), we use the mean to analyse the data. The mean of the given data is 10 828 hours.
- (d) The mean of this batch of data is greater than 10 000 hours. So, from these data, there is no reason to doubt the manufacturer’s claim.

7

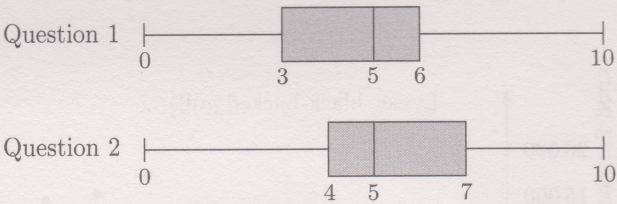
- (a) The problem is one of summarizing.
- (b) A suitable precise question is: ‘What is the mean number of cars per respondent?’
- (c) In the light of the answer to part (b), we use the mean to analyse the data. The mean number of cars per respondent is 1.51.
- (d) Members of the organization seem to have on average one and a half cars in their households. But it would be unwise to generalize this beyond the members of the motoring organization to the general public since the respondents are from a self-selecting group (for example, very few people who do not have a car belong to motoring organizations).

8

- (a) The question is one of comparing.
- (b) A suitable precise question is: ‘What is the ratio of the two battery lifetimes at the mean?’
- (c) The means of the two batches of data are 2266 hours and 6069 hours respectively.
- (d) The ratio of the battery lifetimes at the mean is approximately 2.7, so it looks as though the batteries do not last quite as long as the manufacturer claims.

9

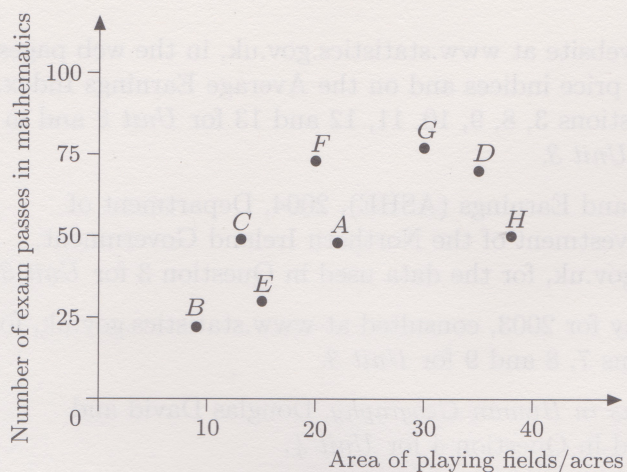
- (a) The question is one of comparing.
- (b) A suitable precise question is: ‘Is there a difference between the distribution of marks scored on the two questions?’
- (c) To answer the question in part (b), we can use means and boxplots. The means are 5 for Question 1 and approximately 5.4 for Question 2. The boxplots for the data are:



- (d) By looking at the boxplots it appears that, although the medians of both sets of marks are the same, the quartiles are higher for Question 2 than for Question 1. This suggests that the top 25% of marks are slightly higher for Question 2 and the bottom 25% are slightly lower for Question 1. Also, the mean for Question 1 is slightly lower than that for Question 2. So it seems that the first question might have been slightly harder.

10

- (a) The problem is one of looking for a relationship.
- (b) A suitable question is: ‘Is there a relationship between the area of the playing fields and the number of exam passes in mathematics?’
- (c) Any relationship should show up on a scatterplot.



- (d) From the scatterplot it seems that there *is* a relationship between the area of the playing fields of a school and the number of exam passes in mathematics. However, in this case the area of a school's playing fields and its number of exam passes in mathematics are probably both related to the size of the school. There may also be other explanations: for example, better-off schools which have larger playing fields may tend to lie in catchment areas containing a higher proportion of well-motivated pupils.

Acknowledgements

The National Statistics website at www.statistics.gov.uk, in the web pages giving data on consumer price indices and on the Average Earnings Index, for the data used in Questions 3, 8, 9, 10, 11, 12 and 13 for *Unit 2* and in Questions 10 and 11 for *Unit 3*.

Annual Survey of Hours and Earnings (ASHE), 2004, Department of Enterprise, Trade and Investment of the Northern Ireland Government, available at www.detini.gov.uk, for the data used in Question 3 for *Unit 3*.

The New Earnings Survey for 2003, consulted at www.statistics.gov.uk, for the data used in Questions 7, 8 and 9 for *Unit 3*.

Smith, D. (1977) *Patterns in Human Geography*, Douglas David and Charles, for the data used in Question 4 for *Unit 4*.

British Medical Journal for the data used in Question 10 for *Unit 4*.

Dyfed Wildlife Trust for the data used in Questions 2, 3, 4, for *Unit 5*.

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Resource Book A Units 1–5

Resource Book B Units 6–9

Resource Book C Units 10–13

Resource Book D Units 14–16

